

A N
INTRODUCTION
T O
PLAIN TRIGONOMETRY,
WITH ITS APPLICATION
T O
HEIGHTS and DISTANCES.

Containing an Explanation of the three Varieties of right angled Triangles, and the four Cases of Oblique, together with a Variety of Questions interspersed by way of Exercise.

By RICHARD COCKRELL, K
Teacher of the Free School, at LARTINGTON.

*Perveniri ad summa, nisi ex principiis, non
potest. Quint.*

DARLINGTON:

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MDCCXCII.

INTRODUCTION
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PLAIN TRIGONOMETRY,

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Containing an Exposition of the three Varieties of right angled
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Variety of Questions and Exercises.



Printed by J. B. Nichols, at the British Museum, and
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MADE IN

THE PREFACE.

THE Number of Publications already extant, treating on TRIGONOMETRY, I am well persuaded occupy no inconsiderable Part of the Bookseller's Shelves. But as Improvement does not always keep Pace with the Time and Labor bestowed on a Subject, so the Treatises already abroad on this Science, do not exclude any Necessity for further Elucidation.

The two following substantial Reasons, are what I lay before the Public as an Apology for sending this Work abroad into the World.

I. That the Books for the most part which contain any Thing of Plain Trigonometry, are Books which treat of one or more of the other mathematical Sciences, as Mensuration, Navigation, Astronomy, &c. and thereby the Price of the Books becomes 5, 6 or perhaps 7s. 6d. Purchase. Now I have by treating of this Branch singly, reduced the Purchase, several Shillings—an Object you know to those in poor Circumstances! And II. The Way in which I have treated this Subject, is entirely new, I mean as to the Plan. And as far as I see,
a but

(but to be sure a Father is apt to judge partially of his own Child) it is so plain and easy that to be comprehended, it need only to be read. I have taught by this Method for nearly 6 Years, and with such Success, that I am induced to lay it open to the World, in hopes that it may facilitate the Acquisition of so useful a Science.

I have in the first Place treated on Practical Geometry, but have used only such Problems as the Work requires; looking upon any other as superfluous.

You have, in the next Place, given a Compendium of Speculative Geometry, in Order to prepare the Learner for a right Understanding of the Demonstrations of the several Rules which are to be found in the Work. Some of the Propositions I have selected from other Authors.

As to the Plan pursued in solving the 3 Varieties &c. of Trigonometry, I shall not speak of here, as a few Observations, by Way of an Advertisement to the Learner, are laid down on the

the Subject and placed before the 1st. Variety, and therefore shall only say, that I have rendered the Application of Trigonometry to Heights and Distances, as useful as possible, by inserting various Kinds of Examples, several of which are borrowed from Dr. HUTTON's excellent Treatise on Mensuration.

In short, I have endeavoured throughout the whole, to be as plain and concise as possible, and indeed too much Pains and Labor cannot be bestowed (to make any Improvement) on the Science of Trigonometry, it is of such general Utility. It may with Propriety be termed the Basis of mathematical Learning. The Surveyor in measuring and laying out his Lands knows the Utility of it. The Mariner daily and hourly experiences convincing Proofs of its salutary Effects. But doth it rest here? No—see it mount up to the Skies to support the Astronomer in his lofty Researches, leads him on to new Discoveries, finds the Magnitude, Distance, Motion, Eclipses &c. of the Heavenly Bodies, and explains the Cause of the Diversity of Color in the Rainbow, to the natural Philosopher.

sopher. Such are the Powers of Trigonometry. It is to this Science that we owe many valuable Discoveries—Discoveries which would perhaps for ever have been numbered with the Arcana of Nature.

Therefore, considering its general Utility, I would advise every one who means to study to any great Length, to get acquainted with the Principles of it, for I am well assured, that no one who is ignorant of them, can make any great Progress in the mixed Mathematics.

Osman-Flat,
Oct. 22d. 1792.

RICHARD COCKREL.

ABBREVIATIONS.

— means Addition, to 2 + 4 means 2 and 4 are to be added together.

— means Subtraction, to 4 — 2 means 4 and 2 are to be taken from 4.

— means Equality, to 4 = 2 + 1 means 4 is equal to 2 added to 1.

X signifies any Angle.

ABBREVIATIONS.

$+$ signifies Addition, so $2 + 4$ means that 2 and 4 are to be added together.

$-$ denotes Subtraction, so $4 - 2$ signifies that 2 is to be taken from 4.

$=$ denotes Equality, so $4 = 3 + 1$ signifies that 4 is equal 3 when added to 1.

$\angle$ signifies any Angle.

PRACTICAL GEOMETRY.

CHAP. I.

SECTION I.

DEFINITIONS.

1. **G**EOMETRY is a Science which treats of extended Quantities. Extention is
1st. Length, 2^{dly}. Length and Breadth, and
3^{dly}. Length, Breadth and Thickness.

2. A Point is indivisible and therefore has no Parts.

3. A Line is generated by the Motion of a Point whether it be straight as A B, or in a curve as C D.

A ————— B

C ————— D

4. Parrallel Lines are such as are equidistant in all their Parts, from each other, as A B, and C D.

A ————— B

C ————— D

A

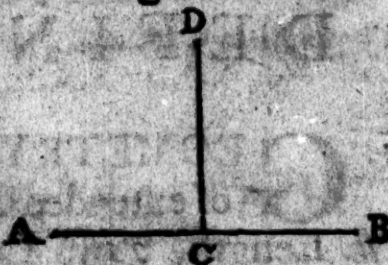
5. Two

PRACTICAL GEOMETRY.

5. Two right Lines as AB , and AC , do, when they meet, make what is called an Angle, as A .



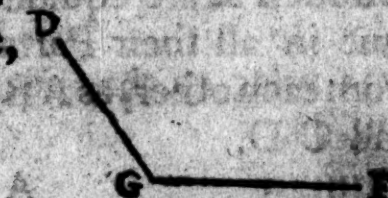
6. When one right Line falls on another right Line, and makes the Angles on each side equal, it is said to be perpendicular to the Line on which it falls; and the Angles are called right Angles. Thus: DC is perpendicular to AB ; and the Angle ACD equal the Angle DCB , is a right Angle.



7. An acute Angle is less than a right Angle, as the Angle CAB .



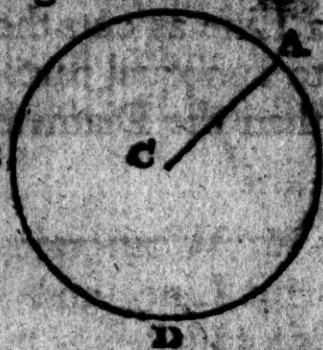
8. An obtuse Angle is greater than a right Angle, as DGF .



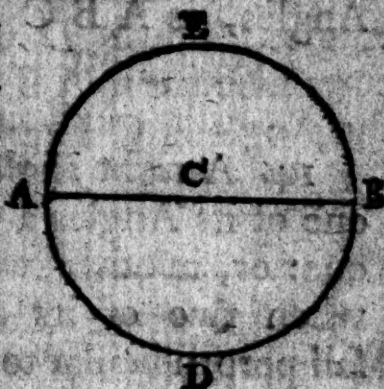
9. A.

PRACTICAL GEOMETRY.

9. A Circle is a regular Figure, and is generated by the Revolution of the Line C A, (round the Centre C) called its Radius; the Line B D A, described by the Point A, in this revolution, is called the Circumference or Periphery of the Circle.



10. A Line passing through the Centre of a Circle, and terminated by the Circumference, is called a Diameter; as A B. It divides the Circle into two equal Parts called Semicircles, as A E B, and A D B.



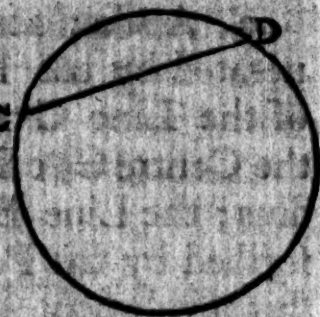
11. A Quadrant is half a Semicircle cut off by a Line perpendicular to the Diameter, as A C D.



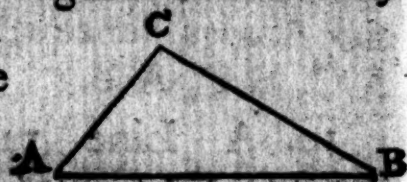
12. A

4 PRACTICAL GEOMETRY.

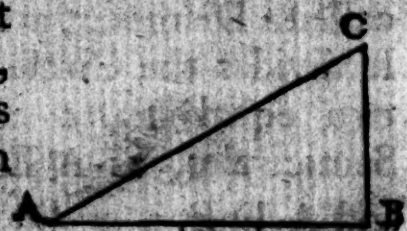
12. A Chord Line is such a Line as cuts the Circle into 2 unequal parts, and is less than the Diameter, as C D.



13. A Triangle is a Figure bounded by three right Lines, and consequently has three Angles, as A B C



14. A right angled Triangle is such as has one of its Angles a right one: or, in other words, when two of its Sides fall perpendicular to each other, as A B C.



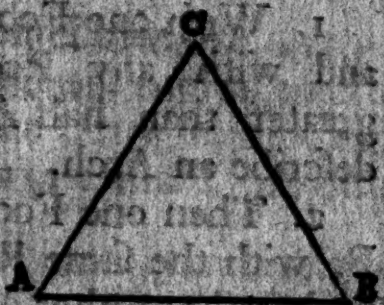
15. An Ifofcales Triangle is such as has two of its Sides equal, as D F = E F.



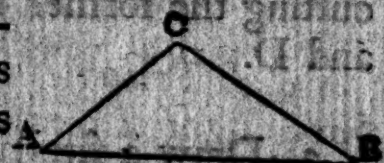
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PRACTICAL GEOMETRY.

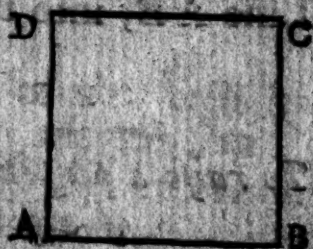
16. An equilateral Triangle hath all its Sides and Angles equal, as $A C = B C = A B$.



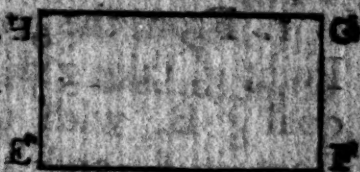
17. A Scalenous Triangle hath all its Sides and Angles unequal, as $A C, B C, A B$.



18. A Square has 4 equal Sides and all perpendicular to each other, as $A B C D$.



19. A Rectangle or Parallelogram is a four sided Figure whose opposite Sides are equal, and as in the Square, the Sides are perpendicular to each other, as $E F G H$.



PROBLEM I.

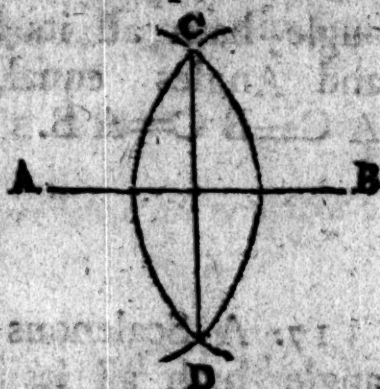
To bisect a given right Line, or divide it into two equal Parts.

1. With

6 PRACTICAL GEOMETRY.

1. With one Foot of the Compasses in A, and with any Radius greater than half A B, describe an Arch.

2. Then one Foot in B, with the same Radius describe another Arch, cutting the former in C and D.

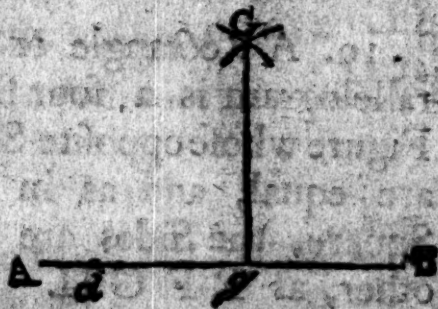


3. Draw a straight Line through C and D, the Points of Intersection, which will cut the Line A B, into two equal Parts

P R O B L E M. 2.

To raise a perpendicular, at or near the Middle of a given right Line A B.

1. From the given Point g, take g d, equal g B, with one Foot in d, and any Radius greater than g, describe an Arch.



2. With the same Radius and one Foot in B, describe another Arch, cutting the Former in G.

3. From

PRACTICAL GEOMETRY. 73

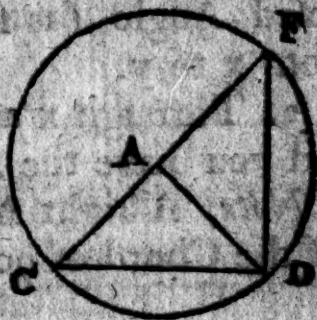
3. From the point of Intersection G, draw the Line G g, the perpendicular required.

PROBLEM 3.

To draw a perpendicular, at the end of a given right Line.

1. With one Foot in D open the Compasses to any Radius, as D A.

2. With A as a Centre describe such a Circle as will pass through the point from whence the perpendicular is to be raised, as at D, let it also cut the Line in any other part as at C.



3. Through the Point of Intersection C and the Centre A, draw the Diameter C F.

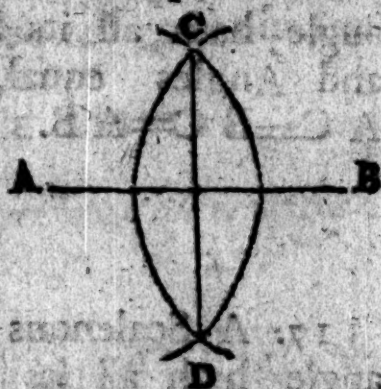
4. Draw F D, which will be the perpendicular required.

Problem 4.

6 PRACTICAL GEOMETRY.

1. With one Foot of the Compasses in A, and with any Radius greater than half A B, describe an Arch.

2. Then one Foot in B, with the same Radius describe another Arch, cutting the former in C and D.



3. Draw a straight Line through C and D, the Points of Intersection, which will cut the Line A B, into two equal Parts

P R O B L E M. 2.

To raise a perpendicular, at or near the Middle of a given right Line A B.

1. From the given Point g, take g d, equal g B, with one Foot in d, and any Radius greater than g, describe an Arch.



2. With the same Radius and one Foot in B, describe another Arch, cutting the Former in G.

3. From

PRACTICAL GEOMETRY.

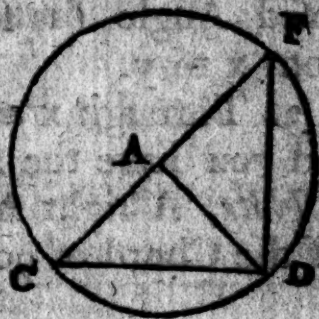
3. From the point of Intersection G, draw the Line G g, the perpendicular required.

P R O B L E M 3.

To draw a perpendicular, at the end of a given right Line.

1. With one Foot in D open the Compasses to any Radius, as D A.

2. With A as a Centre describe such a Circle as will pass through the point from whence the perpendicular is to be raised, as at D, let it also cut the Line in any other part as at C.



3. Through the Point of Intersection C and the Centre A, draw the Diameter C F.

4. Draw F D, which will be the perpendicular required.

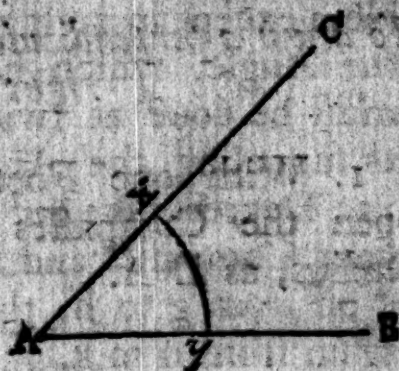
Problem 4.

8. PRACTICAL GEOMETRY.

PROBLEM 4.

* *How to make an acute Angle of any No. of Deg. i. e. less than 90.*

1. With the Chord of 60° as a Radius in your Compasses, and one Foot in A describe the Arch x y.



2. From the Line of Chords take the proposed Angle (as suppose 60) and set it off from x to y.

3. Through x and y draw A C, and A B, and you have the Angle required.

Note, A right Angle is produced by laying off the Chord of 90 Degrees.

Problem 5.

* *Note.* The quantity of Angles is determined by the parts of the Circumference of a Circle, which are contained between the Lines that form the Angle, when the angular Point is considered as the Centre, and the Lines as Radiuses of the Circle. The Circumference of every Circle, is supposed to be divided into 360 equal Parts called Degrees, and these Degrees into 60 Minutes, &c. Now by comparing Def. 6, and 11, it is evident, that a right Angle contains just a Quadrant, that is 90 Degrees: An acute Angle must therefore contain less than 90; and an obtuse one more.

PRACTICAL GEOMETRY.

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PROBLEM 5.

How to draw an obtuse Angle.

This differs very little from the last Problem except in laying off the Angle, which here must be done at twice, as supposing the Angle 120 Degrees take 60° in your Compasses and lay it off from g to h, then again from h to k.

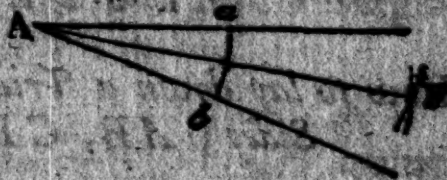


2. Draw the Lines A C and A D, and you have the obtuse Angle required.

PROBLEM 6.

To bisect a right lined Angle.

1. With any convenient Radius in your Compasses and one Foot in A, describe an Arch a b.



2. With one Foot of your Compasses in a, and then in b, describe two Arches cutting each other in F.

B

3. Draw

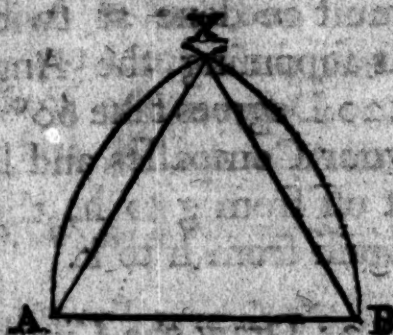
3. Draw F A which will cut a b into two equal parts.

PROBLEM 7.

How to describe an equilateral Triangle upon a given right Line.

1. With the Radius A B in your Compasses and one Foot in B, as a Centre, describe an Arch.

2. Then one Foot in A, with the same Radius describe another Arch, cutting the Former in X.



3. Draw two right Lines from A and B to the point of Interfection X, and the Triangle is completed.

PROBLEM 8.

How to lay down a Triangle from three given Sides ; A B, C D, and E F.

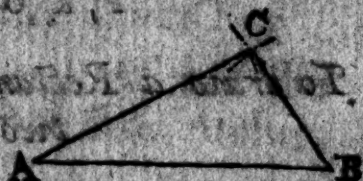
1. Draw a Line and set off thereon A B,



2. With

PRACTICAL GEOMETRY.

2. With C D in your Compasses, and one Foot in B , describe a small Arch.



3. With E F taken in your Compasses, and one Foot in A , describe another Arch cutting the former one.

4. From the Point of Intersection C , draw CA , and CB , and it is done.

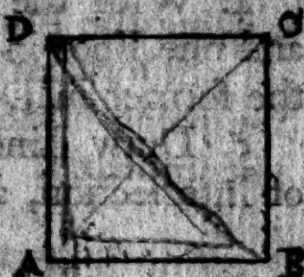
PROBLEM 9.

To form a Square, upon a given right Line.

1. Raise a perpendicular at the end of the given Line B , by Problem 3.

2. Make BC , equal AB .

3. With the same Length in your Compasses, and one Foot in C , draw an Arch x .



4. With the same extent and one Foot in A , describe another Arch, cutting the other in D .

5. Draw CD and AD , which completes the Square.

Problem 10.

P R O B L E M 10.

To draw a Rectangle or Parallelogram from two given Sides.

1. Erect a perpen- 1 —————
dicular at the End of 2 —————
the given Line as at B, by Problem 3.

2. Set off thereon the D —————
Length of the shorter Side,
as B C.

3. With the Distance
of the longer Side in your A ————— B
Compasses and one Foot in C, describe an
Arch.

4. With the Distance of the shorter Side
and one Foot in A, describe an Arch, cutting
the former in D.

5. Draw Lines from C and A to the Point
of Intersection, and it is done.

P R O B L E M 11.

*Through a given Point, to draw a right Line
parallel to another given right Line.*

1. From the given Point A, draw a Line
at pleasure to meet R ————— S
the given right Line
P Q at some Point,
suppose at B.



2. Make

2. Make $BC = AB$.

3. With the Centres C and A, and the Radius BC, or AB, make two little Arches, crossing one another in D.

4. Through the Points A and D, draw the right Line RS, and it is done.

SPECULATIVE

SPECULATIVE GEOMETRY.

With the Centres C and A, and the Radii B C, or A B, make two little Arches,

AXIOMS.

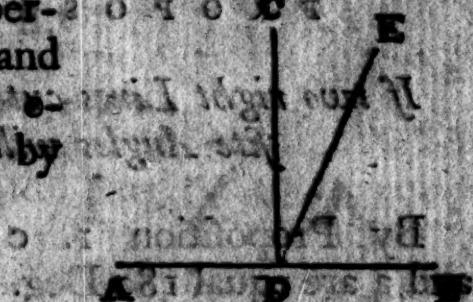
1. **I**F equal Quantities be added to equal Quantities, their Sums will be equal.
2. If equal Quantities be taken from equal Quantities, the Remainders will be equal.
3. If unequal Quantities be added to unequal Quantities, their Sums will be unequal.
4. If unequal Quantities be taken from unequal Quantities, the Remainders will be unequal.
5. Two Quantities respectively equal to a third, are equal to each other.
6. If Lines and Angles are equal, they will not exceed when laid upon each other, but will mutually agree.
7. The Whole is greater than its Part.
8. The Whole is equal to all its Parts taken together.

PROPOSITION. I.

If a right Line stand upon another right Line, It makes either two right Angles, or two Angles equal to two right ones.

1. If

1. If $C D$ be perpendicular, $A D C$ and $C D B$, will be each equal to a right Angle by Definition 6.



2. And if the Line is not perpendicular but $C D E$, then $A D E$ and $E D B$, are equal to $A D C$ and $C D B$, for $C D E$, which is taken from one right Angle is added to another, and therefore, the whole must be the same, $Q E D$.

C O R O L L A R Y.

Hence it appears that all the Angles, that can be made, at any Point, on the same Side of a right Line, are equal to two right Angles. For, suppose any number of Lines drawn from the Point D , on the same side the right Line $A B$, and the Angles formed by those Lines will be Parts of the two right Angles $A D C$ and $C D B$. But, all those Angles taken together will equal all the Parts of the 2 right Angles; and therefore, by Axiom 3, equal two right Angles.

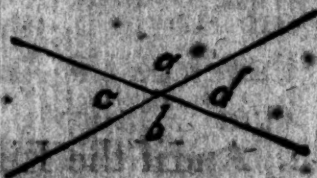
Proposition 2.

10. SPECULATIVE GEOMETRY.

PROPOSITION 2.

If two right Lines cut each other, the opposite Angles will be equal.

By Proposition 1. c and a are equal 180° Deg. and c and b are equal 180° Deg. by the same. But c is common to both, therefore a and b must be equal. Q E D.

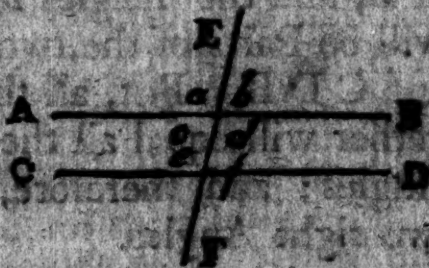


PROPOSITION 3.

If two Parallel Lines are cut by a right Line, it shall make the external Angle equal to the internal Angle, laying on the same side; the alternate Angles equal; and the two internal Angles laying on the same side, are equal to two right Angles.

DEMONSTRATION.

1. By Def. 4. Sect. 1. The Line AB , will have the same Inclination to the Line EF , as CD has, otherwise



they

they cannot be parallel, therefore the Angle b is equal to the Angle f, and the Angle a equal to the Angle e.

2. The alternate Angles are equal, thus: I have already proved the Angle f to be equal to the Angle b, and by Prop. 2. c is equal to b, therefore the Angle c is equal to the Angle f, (per. Axiom 5.)

3. The two internal Angles d and f are equal to two right Ones, thus: the Angles a and b are equal to two right Angles, per Prop. 1. The Angle a, equal to the Angle d, by Prop. 2; and the Angle b is equal to the Angle f, by the 2d. of this; therefore d being equal to a, and f equal to b, d and f are equal to a and b, and consequently equal to two right Angles.

Q E D.

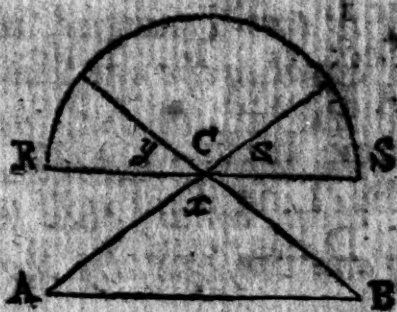
PROPOSITION. 4.

The three Angles of any Triangle are always equal to two right Ones, or 180 Degrees.

DEMONSTRATION.

Draw R S, parallel to A B, through C: then by Corol. to Prop. 1. y, C and z, are equal to two right Angles, but the Angle C is equal to the Angle x, by

C



Prop. 2.

18 SPECULATIVE GEOMETRY.

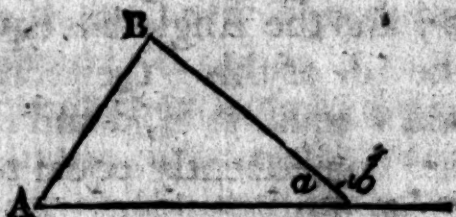
Prop. 2. and the Angles y and z equal to the Angles B and A , respectively, by Prop. 3. therefore the three Angles A , B and x , are equal to the three Angles y , C and z , and consequently equal to two right Angles. Q E D.

PROPOSITION 5.

The external Angle of a Triangle, will always be equal to the two internal opposite ones.

DEMONSTRATION.

By Prop. 1. a and b are equal to two right Angles, which are also equal to the 3 Angles of the Tri-



angle. But a is equal to the one by Inspection; therefore b is equal to the other two, A and B , taken together. Q E D.

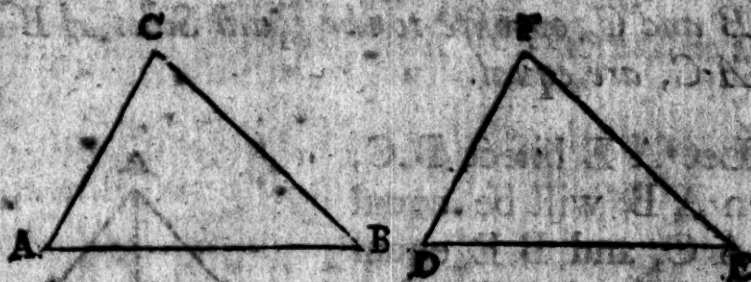
PROPOSITION 6.

If in two Triangles, the one Side of the one, be equal to the one Side of the other, and another Side of the one, be equal to another Side of the other, and the Angles contained between them, equal; then shall their Bases be equal, and consequently the whole Triangle.

Demonstration.

SPECULATIVE GEOMETRY. 19

DEMONSTRATION.



Suppose the one Triangle to be laid on the other, i. e. the Point F on the Point C, now as A C and C B, are equal to D F and F E, respectively, and the Angle contained between them also equal, the Lines D F and F E, will coincide with A C and C B, (per Axiom 6.) and consequently A B, with D E, and as each Triangle occupies exactly the same space, they are equal. Q E D.

C O R O L L A R Y.

From hence it will follow, that if two Triangles have one Side of one, equal to one Side of the other, and the Angles at each End respectively equal, the Triangles are equal in all Respects.

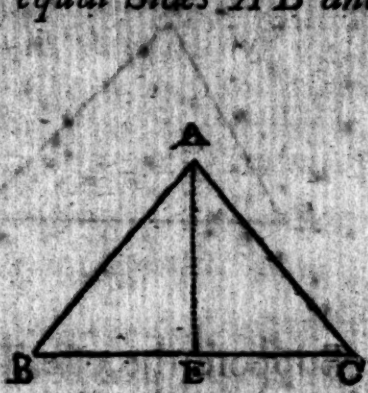
Proposition 7.

SPECULATIVE GEOMETRY.

PROPOSITION 7.

In an Iſoſceles Triangle ABC , the Angles B and C , oppoſite to the equal Sides AB and AC , are equal.

Let AE biſect BC , then AB will be equal to AC , and BE equal to EC ; and AE being common to both the Triangles AEB and AEC , it follows that the Angles at B and C muſt be equal, by the laſt part of the laſt Prop. It follows alſo, that AE is perpendicular to BC , and that it biſects the $\angle BAC$.



PROPOSITION 8.

If the three Sides of two Triangles are equal, the Angles oppoſite to thoſe Sides will be equal.

The truth of this Propoſition is evident by the two included Triangles in the laſt Propoſition, for they have their reſpective Sides equal, viz. BE equal EC , and BA equal AC , and AE common to both Triangles; and it is there proved, that the Angles oppoſite to thoſe equal Sides are equal. QED . The

SPECULATIVE GEOMETRY. 21

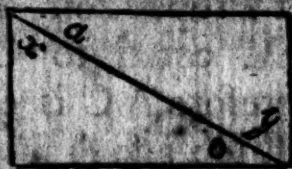
N. B. The Converse of this proposition holds not true, for the Angles of two Triangles may be equal, and their opposite subtending Sides unequal, as it will hereafter appear.

Hence it follows, that two Angles mutually equilateral, are also mutually equiangular.

PROPOSITION 9.

The opposite Sides and Angles of Parallelograms are equal, and the Diagonal A C, divides it in the Middle.

In the Parallelogram A B C D, the opposite Angles a and o are equal, for a is equal to o, because alternal Angles by Prop. 3. and x is equal to y, by Prop. 3. therefore the Angle a is equal to Angle o; again, the opposite Sides B A and C D are equal; also B C and A D are equal, for in the Triangles A B C and A D C, the Angles x and o are equal to the Angles y and a, and the Side A C common to both Triangles; therefore A B C is equal to the Triangle A D C by Corol



SPECULATIVE GEOMETRY.

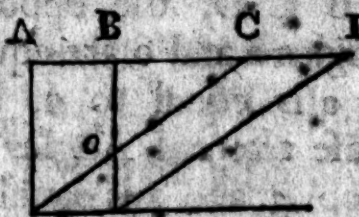
Corol. Prop. 6. consequently the Sides $A D$ and $B C$ as also $A B$ and $D C$, must be equal,

C O R O L L A R Y.

The Line $A E$, drawn to the opposite Angles, which is called the Diagonal, divides the Parallelogram into two equal parts.

P R O P O S I T I O N 10.

Parallelograms on the same Base, and between the same Parallels, are equal: Thus the Parallelogram $A E M B$, is equal to the Parallelogram $C E M D$, for since by the last Proposition, $A B$ is equal to $E M$, and $E M$ equal to $C D$, therefore $A C$ is equal to $B D$; also $A E$ equal to $B M$, and $E C$ equal to $M D$, therefore the Triangles $C E M$



$A E$ and $D M B$ must be equal by Prop. 8 and 6. therefore taking away $B o C$, from both Triangles, there will remain the Space $A E o B$, equal to $C o M D$, by Axiom 2. then, by adding $E o M$ to both these Spaces, you will have the Parallelogram $A E M B$, equal to the Parallelogram $C E M D$ by Axiom 1. $Q E D$.

Corol.

SPECULATIVE GEOMETRY. *

COROLLARY.

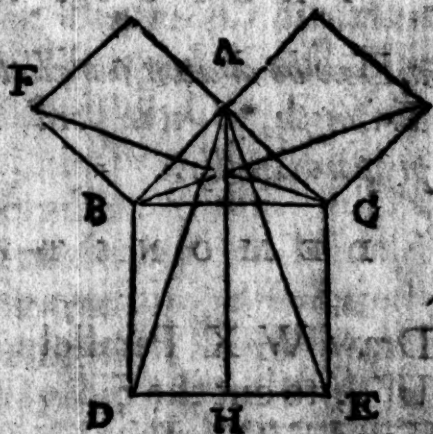
Hence Triangles upon the same Base, and between the same Parallels are equal, because they are Halfs of Parallelograms described on the same Base, and between the same Parallels.

PROPOSITION II.

In every right angled Triangle, the Square of the Hypotenuse (or Side subtending the right Angle,) is equal to the Sum of the Squares of the other two Sides.

DEMONSTRATION.

Draw the Line A H, Parallel to B D, and join A D and A E, the Angle D B C, is equal to the Angle F B A, and add the Angle A B C to both, then is the Angle A B D equal to the Angle F B C, moreover A B is equal to F B, and B D equal to B C, by the Definition of a Square



SPECULATIVE GEOMETRY.

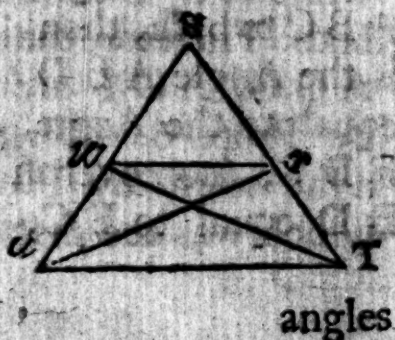
Square; therefore is the Triangle $A B D$, equal to the Triangle $F B C$, by Prop. 6. But the Parallelogram $B H$, is equal to twice the Triangle $A B D$, and the Parallelogram $C H$, is equal to twice the Triangle $A C E$, by Corol. Prop. 10. also the Square $A F$, is double the Triangle $F B C$, and the Square $A G$, double the Triangle $G C D$. But the Triangle $F B C$, is equal to the Triangle $A B D$. And the Triangle $G C B$, is equal to the Triangle $A C E$; therefore the Square $F A$, is equal to the Rectangle $B H$, and $A G$ equal to $C H$, and by consequence the Squares $A F$ and $A G$, are equal to the Square $B D E C$. $Q E D$.

P R O P O S I T I O N 12.

If a Line be drawn parallel to any Side of a given Triangle, it will cut the other two Sides proportionally.

D E M O N S T R A T I O N.

Draw $W X$ Parallel to $U T$, to cut the Sides $S U$ and $S T$ in W , X ; then $S W : S U :: S X : S T$; draw $W T$ and $U X$, making the Tri-



SPECULATIVE GEOMETRY. 135

angles UWX and TXW ; therefore it will be, as $SWX:WUX::SW:WU$, and, as $SWX:WUX$ (the Triangles UWX and TXW being equal) $\therefore SX:XT$; then throw away the two first Terms SWX , WUX , and it will be, as $SW:WU::SX:XT$. Q.E.D.

PROPOSITION 13.

If in a Circle, a right Line drawn through the Centre bisects another, not drawn through the Centre, it will cut it perpendicularly, and if it cut perpendicularly it will bisect it.

DEMONSTRATION.

Part I. From the Centre A , let there be drawn AC , AF the Triangles X and Z are equilateral to each other. For CO , FO are by the Hypothesis equal, and AC , AF are so, because drawn from the Centre; while AO is common to both. Therefore the Angles AOC , AOF are equal by Prop. 6. Therefore right ones, by Prop. 1. which was the first part.



D

2. Because

2. Because by the Hypothesis AOC , AOF are right Angles; AC Square shall be equal to the Squares of AO , OC together, by Prop 11; and AF Square, equal to the Squares of AO , OF together. Seeing therefore the Squares of AC , AF are equal, the Squares of AO , OC together will be also equal to the Squares of AO , OF together. Wherefore taking away the common Square AO , the Squares of OC , OF remain equal. And therefore the right Lines OC , OF are equal. Which was the other part.



TRIGONOMETRY.

(37)

TRIGONOMETRY.

DEFINITIONS.

Trigonometry is that Part of the mixed Mathematics, that teaches us to calculate the Measures and Proportions of the several Parts of Triangles. The Parts of a plain Triangle are six: three Sides and three Angles. These are therefore the subject of Plain Trigonometry. And in all our Enquiries three Things (either all the Sides, or Sides and Angles,) are given to find a fourth, (Side or Angle.)

1. **E**VERY Circle whether great or small is conceived to be divided into 360 equal Parts called Degrees, these Degrees are subdivided again into 60 equal Parts called Minutes, these again into Seconds &c. See *Note to Prob. 4. Sect. 1.*

2. A plain Triangle is projected on a plain or flat Superficies, and therefore its Sides are right Lines.

3. A

3. A plain Triangle is either right angled or oblique angled.

4. A right Angle is such as measures just 90 Degrees, therefore a Triangle which has one of its Angles equal to 90 Degrees is called a right angled Triangle.

5. An oblique angled Triangle, is such as has no right angle.

6. In right lined Trigonometry the Sum of the Angles of any Triangle, is always a Semicircle or 180 Degr. (Prop. 4. Sect. 2.) therefore in a right angled Triangle, one Angle (i. e. the right one) is always understood to be given; and if another Angle is given, in course, by subtracting it from 90 Degr. or adding it to the right Angle, and subtracting their Sums from 180 Degr. it will leave the third Angle.

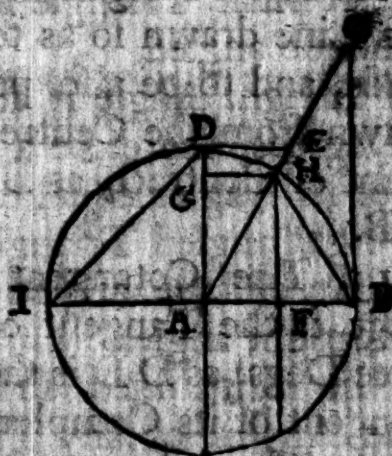
7. In a right angled Triangle, the Side opposite to the right Angle, is called the Hypotenuse and the two Sides containing the right Angle, are called the Base and Perpendicular.

8. The Complement of any Number of Degr. is what they want of 90 Degr. as H D is the complement of B H.

9. The

9. The Supplement of any Number of Degs. is what they want of 180. as $H D I$ is supplement of $B H$.

10. The Sine of any Arch, is a Line drawn from one End of it perpendicular to a Diameter that passes through the other End, as $H F$ is the Sine of $H B$.



11. The Co-sine of any Arch, is the Sine of its Complement as $H G$ is the Co-sine of the Arch $H B$, or the Sine of its Complement $H D$.

12. The versed Sine of any Number of Degs. is that Space which is contained between the Sine and Circumference measured upon a Line passing through the Centre, as $F B$ is the versed Sine of $H B$.

13. The Covered Sine of any Number of Degs. is the versed Sine of the Complement of those Degrees as $G D$, is the covered Sine of $B H$, or versed Sine of the Complement of those Degrees.

14. The

14. The *Tangent of any Number of Degr. is a Line drawn so as to meet it at one of its Ends, and to be also perpendicular to a Line drawn from the Centre of the Circle to the Point of Contract, as BC is the Tangent of HB .

15. The Cotangent of any Number of Degr. is the Tangent of the Complement of those Degr. as DE is the Cotangent of BH , or Tangent of its Complement HD .

16. The † Secant of any Number of Degr. is a Line drawn from the Centre of the Circle through one end of it, until it cuts the Tangent of those Degr. as AC is the Secant of BH .

* Tangent comes from the Latin Word *tango*, to touch, and the Line is so called, because it just touches the Periphery of the Circle.

† Secant comes from the Latin Word *seco*, to cut, and is so called, because it cuts the Circumference of the Circle.

ADVERTISEMENT.

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BEFORE we proceed to the Solution of the Questions, it will be necessary that the Learner be acquainted with the Plan we shall pursue.

Plain Trigonometry naturally divides itself into two principal Parts: viz. I. Right angled Plain Trigonometry and II. Oblique.

Right angled Trigonometry is usually divided into six Cases, according as the Data and Quæsitæ, (the Things given and Things required,) differ. But as each of these Cases admits of a Solution in three various Methods, we have to avoid Perplexity, divided the whole Doctrine of right angled Triangles into three Varieties, and have solved all the Cases separately in each of them.

These Varieties are determined from the Side that is made Radius; thus:

I. In the first Variety the Hypothenuſe is made Radius, and consequently the Legs will be the Sines of their opposite Angles.

II. In

II. In the second the Base is supposed Radius, and then the Perpendicular becomes Tangent, and Hypothenuſe Secant.

III. In the third the Perpendicular is conſidered as Radius and the Hypothenuſe will be Secant and Baſe Tangent.

In order to illuſtrate the Proportions and aſſiſt the Learner to diſcover the true Relation, one Part of the Triangle has to another, three Plates are added, adapted to the 3 Varieties. In theſe the Triangle is diſſected and each Part, whether Side or Angle, placed oppoſite its correſponding Part, with the proper term affixed to both. Thus the Learner needs only turn to the Plate and conſider what Parts he knows, and what he wiſhes to diſcover, and he will inſtantly ſee how his proportion is to be formed. But more of this when we come to the Examples.

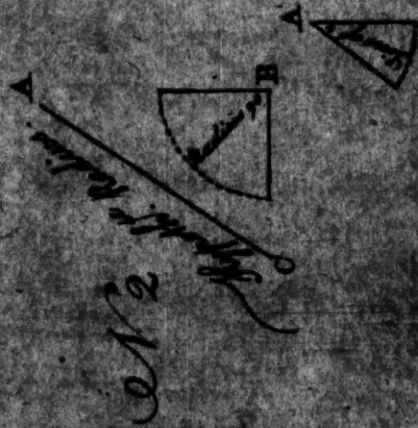
Right

The first Variety Hypothenuse Radius

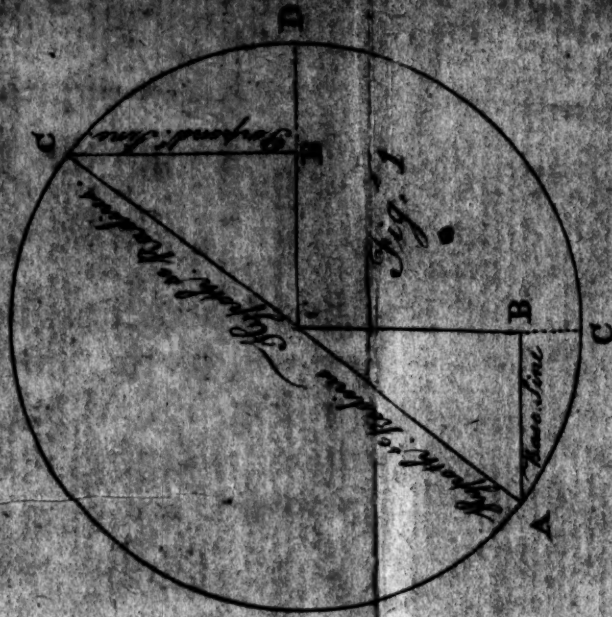
Plate 1

To find Page 33

N^o 1



N^o 3



Richardson's

Right angled Triangles.

V A R I E T Y I.

Hypothenuſe Radius.

IF the Hypothenuſe be made Radius, each Leg will be the Sine of its oppoſite Angle. Plate 1. Fig. 1. Where it is evident (from Definition 7) that A B is the Sine of the Arch A C, and B C the Sine of the Arch C D,

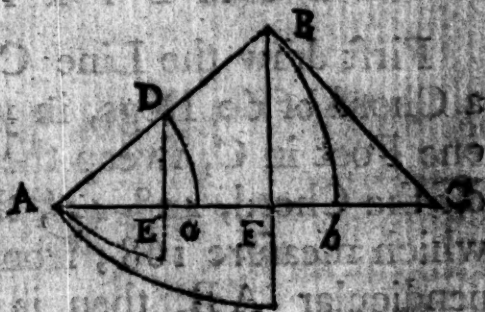
*Rule * As any one Side, is to the Sine of its oppoſite Angle, ſo is any other Side, to the Sine of its oppoſite Angle; and vice verſa.*

E

In

* D E M O N S T R A T I O N.

Let A B C be a Triangle: From D and B, let fall the Perpendicular D E and B F. Now D a and B b being deſcribed with the Radii A D and A B, ſhew that D E and B F are Sines of their oppoſite Angles (by Def. 7.) and ſo of A F and A E. Then per ſimilar Triangles (Prop. 12. Sect. 2.)

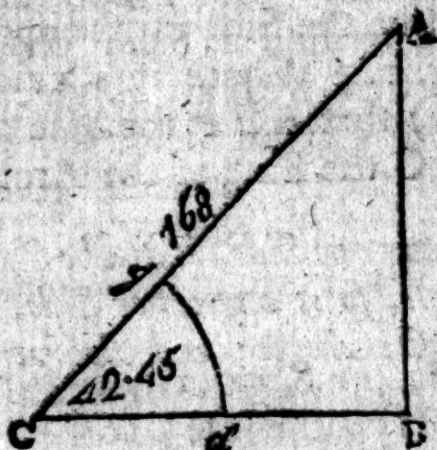


as $AB : BF :: AD : DE$

and

as $AB : AF :: AD : AE$ &c. Q E D.

In the right angled Triangle ABC are given, the Hypothenufe AC 168, and the Angle C $42^{\circ}45'$ to find the Perpendicular.



GEOMETRICALLY.

First draw the Line CB at Pleasure, with a Chord of 60 Degs, in your Compasses, and one Foot in C , sweep the Arch dg , and lay off the Chord, $42^{\circ}45'$, through g draw CA , which measure 168; from A , let fall the Perpendicular AB , then is the Triangle constructed geometrically, the Perpendicular measuring on a Scale of equal Parts 114, and Base BC 115.1.

Logarithmically

LOGARITHMETICALLY.

You have the Hypothenuſe and all the Angles given, but neither the Perpendicular nor Baſe. Now turn to Plate 1, and you will ſoon diſcover how your Proportion will run. Remember, to find a Side, you muſt begin your Proportion with an Angle, which muſt be oppoſite to a given Side; therefore ſeek in the Plate for the Hypothenuſe given, and you will find it to be the Sine of 90 Degs. which muſt be the firſt Term in the Proportion, thus:

As Radius, or Sine of 90° 10.000000
Is to the Hypothenuſe 168 2.225309

Next look for the other given Angle C, and you will find the Perpendicular to be the Sine of it, which will finiſh the Proportion.

So is the Sine of $\angle C$ 42° 45' 9.831742

To the Perpend. 114 2.057051

Again, to find the Baſe, you may uſe the former Part of the laſt Proportion, if you pleaſe, or the Latter either, it matters not which;

which †; then look for the required Side AB which you will find No. 3. with the Angle A opposite to it, therefore your second Proportion will be,

As Radius, or Sine of 90°	10.000000
Is to the Hypoth. 168	2.225309
So is Sine of the $\angle A$ $47^\circ 15'$	9.865887
To the Base 124	<u>2.091196</u>

or

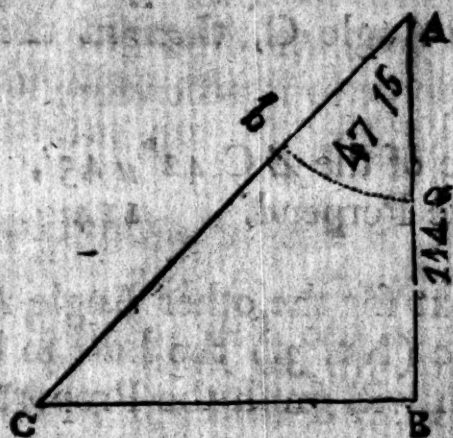
As the Sine of the $\angle C$ $42^\circ 45'$	9.831742
Is to the Perpend. 114	2.057051
So is the Sine of the $\angle A$ $47^\circ 15'$	9.865887
To the Base 124	<u>2.091196</u>

Question 2. Given in the right angled Triangle ABC, the Perpendicular BA 114 and the Angle C $42^\circ 45'$; required the Hypotenuse and Base.

Geometrically.

† That taking either Part of the Proportion makes no Alteration in the Solution will appear evident, when it is considered, that as any given Side is to the Sine of its opposite Angle, so is any other Side to the Sine of its opposite Angle, and *vice versa*.

TRIGONOMETRY.



GEOMETRICALLY.

In the first place draw the Line AB , and set off 114 thereon, then raise BC perpendicular thereunto; next with the Chord of 60° in your Compasses, sweep the Arch $a\ b$ and lay off $47^\circ 15'$, through b produce the Line AC , which completes the Triangle.

N. B. If AC and CB be applied to the Scale of equal Parts, it will give their respective Lengths.

LOGARITHMETICALLY.

You have the perpendicular and Angles given to find the Hypothenuse and Base. Turn to Plate 1, and seek for the Perpendicular, which

which you will find (No. 1) to be the Sine of its opposite Angle C, then in the first place it will be,

As Sine of the $\angle C$ $42^{\circ} 45'$	9.831742
Is to the Perpend. 114	2.057051

Look next for the other Angle A, and you will observe (No. 3.) the Base to be the Sine of it, which will finish the Proportion.

So is Sine of the $\angle A$ $47^{\circ} 15'$	9.865887
To the Base 124	2.091196

Again, to find the Hypothenufe, you may make use either of the former or latter Part of the last Proportion, and looking in the Plate for the required Side C A, you will find it (No. 3.) with the Radius opposite to it, consequently you proceed, thus:

As Sine $\angle C$ $42^{\circ} 45'$	9.831742
Is to the Perpend. 114	2.057051
So is Radius 90°	10.000000
To the Hypoth. 168	2.225309

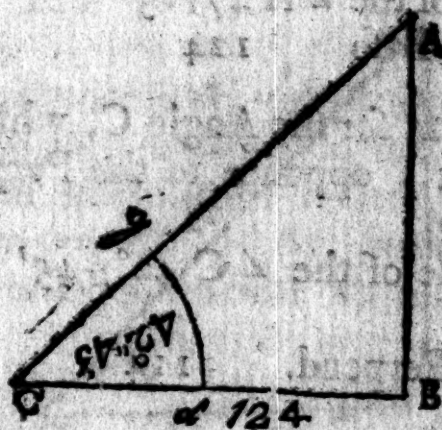
As

TRIGONOMETRY.

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or
 As Sine of the $\angle A$ $47^{\circ} 15'$ 9.865887
 Is to the Base 124 2.091196
 So is Radius 90 10.000000
 To the Hypoth. 2.225309

Question 3. Given in the right angled Triangle ABC , the Base BC 124 and $\angle C$ $42^{\circ} 45'$ required the Hypothenufe and Perpendicular.



GEOMETRICALLY.

Draw the Base BC , and lay off thereon 124 from a Scale of equal Parts; next on the Point B , raise the Perpendicular BA , then with the Sweep of 60° in your Compasses describe the Arch

TRIGONOMETRY.

Arch dg, and lay off thereon $42^{\circ} 45'$ from the Line of Chords from d to g, join CA produced through g which completes the Triangle.

LOGARITHMICALLY.

You have the Base and Angle C given, but neither the Hypothenuſe nor Perpendicular, therefore turn to Plate 1, and find which Angle the Baſe is the Sine of, which you will find (No. 3) to be the Angle A $47^{\circ} 15'$. The two firſt Terms of your Proportion will be as follows.

As Sine of the $\angle A$ $47^{\circ} 15'$	9.865887
Is to the Baſe 124	2.091196

Look next for the Angle C, which you will find (No. 1.) oppoſite to the Perpendicular; therefore,

So is Sine of the $\angle C$ $42^{\circ} 45'$	9.831742
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To the Perpend. 114	2.057051
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Again to find the Hypothenuſe;

As Sine of the $\angle C$ $42^{\circ} 45'$	9.831742
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Is to the Perpend. 114	2.057051
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So is Radius 90	10.000000
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To the Hypoth. 168	2.225309
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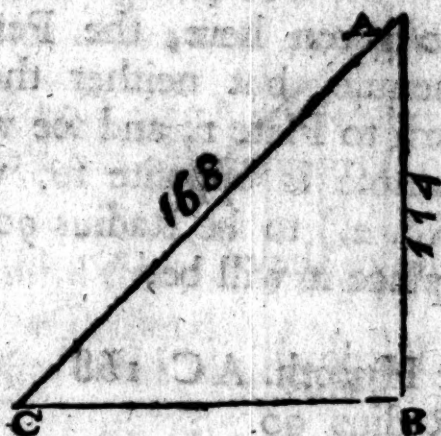
As

TRIGONOMETRY.

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or		
As Sine of the $\angle A$	$47^{\circ} 15'$	9.865887
Is to the Base	124	2.091196
So is Radius	90	10.000000
To the Hypoth.		168
		2.225309

Question 4. Given in the right angled Triangle ABC, the Hypothenufe AC 168 and Perpendicular BA 114 to find the Angles and Base.



GEOMETRICALLY.

Draw the Perpendicular AB and lay off thereon 114, then from the Point B, draw BC perpendicular

F

perpendicular

perpendicular to AB ; next take the Length of the Hypothenufe AC in your Compasses, and one Foot in A , as a Centre, with the other cut the Line BC , as in C , then a Line drawn from A to C completes the Triangle.

N. B. With the Chord of 60° in your Compasses and one Foot in C , describe the Arch dg , which measured in your Compasses and applied to the Line of Chords will give its Length.

LOGARITHMETICALLY.

You have given here, the Perpendicular and Hypothenufe, but neither the Base nor Angles. Turn to Plate 1, and see what Angle the Hypothenufe is opposite to, which you will find (No. 2.) to be Radius 90° therefore in the first place it will be,

As the Hypoth. AC	168	2'225309
Is to Radius	90	10'000000

Look next for the Perpendicular which you will find (No. 1.) the Sine of the Angle C , therefore you proceed, thus:

TRIGONOMETRY.

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So is the Perpend. 114 2.057051

To Sine of the $\angle C$ $42^{\circ} 45'$ 9.831742

Again to find the Base, you must begin with an Angle, as directed in a former Page; therefore look in the Plate for the Base, and you will find it (No. 3.) to be the Sine of the Angle A, your Proportion then will run, thus:

As Radius 90 10.000000

Is to the Hypoth. 168 2.225309

So is Sine of the $\angle A$ $47^{\circ} 15'$ 9.865887

To the Base 124 2.091196

or

As Sine of the $\angle C$ $42^{\circ} 45'$ 9.831742

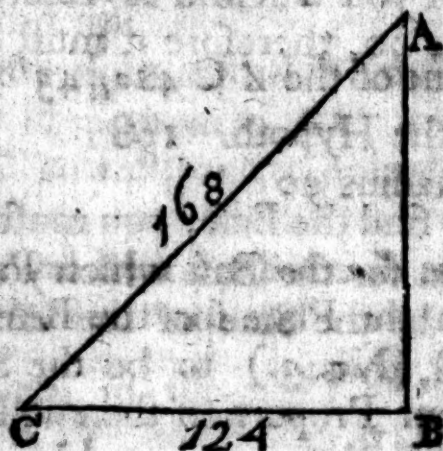
Is to the Perpend. 114 2.057051

So is Sine of the $\angle A$ $47^{\circ} 15'$ 9.865887

To the Base 124 2.091196

Question 5. In the right angled Triangle ABC, are given the Hypothenuse 168 and Base 124 to find the Angles and Perpendicular,

Geometrically



GEOMETRICALLY.

First draw the Line BC and lay off thereon 124; on B erect a Perpendicular, and with an Opening of the Compasses equal to the Length of the Hypothenufe, set one Foot in C and with the other cut BA in A , a Line drawn from C to A completes the Triangle.

N. B. The Angles C and A may be found as directed in the last Question.

LOGARITHMATICALLY.

In this Question you have given the Hypothenufe and Base, but neither the Perpendicular nor Angles. Turn to Plate 1, and see what Angle the Hypothenufe is connected with,

TRIGONOMETRY.

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with, which you will find as before (No. 2.) to be Radius 90, therefore it must be,

As to the Hypoth.	168	2.225309
Is to Radius 90		10.000000

Look then for the Base which you will find (No. 1.) to be the Sine of the Angle A, so you proceed, thus;

So is the Base	124	2.091196
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To Sine of the $\angle A$ $47^{\circ} 15'$		9.865887
--	--	----------

Again, to find the Perpendicular.

As Radius, 90°		10.000000
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Is to the Hypoth.	168	2.225309
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So is Sine of the $\angle C$ $42^{\circ} 45'$		9.831742
---	--	----------

To the Perpend.	114	2.057051
-----------------	-----	----------

OR

As Sine of the $\angle A$ $47^{\circ} 15'$		9.865887
--	--	----------

Is to the Base	124	2.091196
----------------	-----	----------

So is Sine of the $\angle C$ $42^{\circ} 45'$		9.831742
---	--	----------

To the Perpend.	114	2.057051
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Questions

Questions to exercise the first Variety.

1. In the right angled Triangle ABC are given the Hypotenuse AC 100, and the Angle ACB $52^{\circ} 10'$, required the Base and Perpendicular?

Ans. Base 77.9 and Perpend. 62.71.

2. Given, in the right angled Triangle ADG , the Perpendicular AD 75 and the Angle $\angle D$ $33^{\circ} 19'$ to find the Base and Hypotenuse.

Ans. Base 49.3 and Hypoth. 89.7

3. Suppose in the right angled Triangle XYZ , the Base XY is 505, and the Angle Y 40° what are the Perpendicular and Hypotenuse.

Ans. Perpend. 423.8 and Hypoth. 659.2

4. I find in the Triangle BHS , that the Base BH is 87 and Hypotenuse BS 108 quere the Perpendicular and Angles.

Ans. Perp. 63.99, Angle B $30^{\circ} 20'$ & S $53^{\circ} 40'$

5. There

TRIGONOMETRY

5. There is a Triangle whose Hypothenufe is 78 and Perpendicular 60. What are the Angles and Base?

Ans. the Angle opposite the Perp. $50^{\circ} 17'$
and Base 49.84.

6. In the Triangle FCD, are given the Hypothenufe FD 157, and the Angle DFC $33^{\circ} 45'$ quere the Base and Perpendicular?

Ans. the Base 130 and Perpend. 87.2



Base Radius.

IF the Base be made Radius the Perpendicular becomes Tangent and Hypothenufe Secant, See Plate 2, Fig. 2; where it is evident (from Def. 14 and 16) that as the Base AB is made Radius, BC is Tangent, and the Hypothenufe AC Secant.

R U L E *

As the Radius

Is to the Base

So is Tang. of the Angle }
opposite to the Perpend }

To the Perpend.

and

As the Radius

Is to the Base

So is the Secant of the same $\angle$

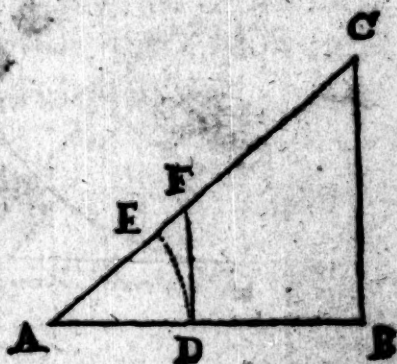
To the Hypothenufe.

And the contrary.

In

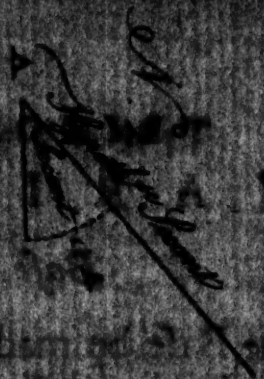
*** D E M O N S T R A T I O N .**

With the Centre A, and any Radius AD, describe an Arch DE, and erect the Perpendicular DF, which, it is evident, will be the Tangent, and AF the Secant of the Arch DE, or Angle A to the Radius AD. And in the Similar Triangles ADF, ABC, it will be AD: AB:: DF: BC:: AF: AC. QED.



The second Variety, Base Radius

Fig. 1



Depend. Tangent

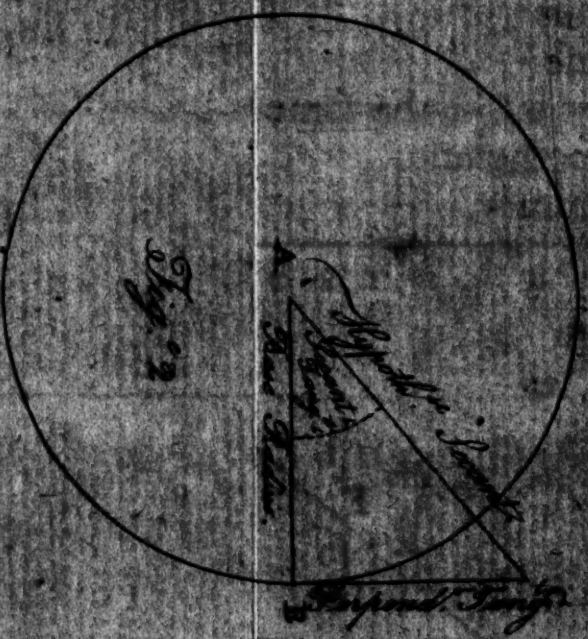
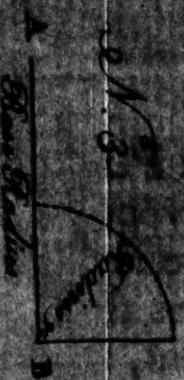
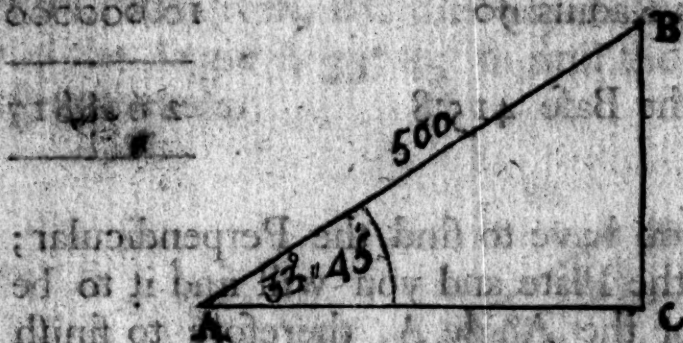


Fig. 2

The third Variety

Question 1. In the Triangle ABC are given the Hypothenufe AC 500 and the Angle. A $33^{\circ} 45'$, to find the Perpendicular and Base.



GEOMETRICALLY.

As the Geometrical Construction in this and the following Variety is much the same as the former, I shall therefore say no more on that Head.

LOGARITHMETICALLY.

You have the Hypothenufe and all the Angles given, but neither the Base nor Perpendicular. Now turn to Plate 2, and you will soon discover how your Proportion will run, for first seek for the Hypothenufe given, and you will find it to be (No. 2.) Secant of the Angle A, therefore you begin, thus:

As Secant A $\angle$ $33^{\circ} 45'$	10.080153
Is to the Hypoth. 500	2.698970
G	As

As the Base is made Radius, and you have to find the Base, the other Part of the Proportion will be,

So is Radius 90	10.000000
To the Base 415.8	<u>2.618817</u>

Next you have to find the Perpendicular; look into the Plate and you will find it to be Tangent of the Angle A, therefore to finish your Proportion, proceed by the latter Part of the last as follows,

As Radius 90	10.000000
Is to the Base 415.8	2.618817
So is the Tang. of $\angle A$ 33 $^{\circ}$ 45'	<u>9.824892</u>
To the Perpend. 277.8	<u>2.443709</u>

Or which will answer the same Purpose,

As Secant of $\angle A$ 33 $^{\circ}$ 45'	10.080153
Is to the Hypoth. 500	2.698970
So is Tang. of $\angle A$ 33 $^{\circ}$ 45'	<u>9.824892</u>
To the Perpend. 277.8	<u>2.443709</u>

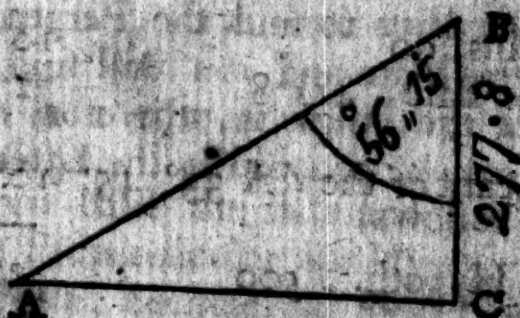
N. B.

TRIGONOMETRY.

51

N. B. The Side made Radius generally comes first in the Proportion, if it is given.

Question 2. In the right angled Triangle ABC, are given, BC the Perpendicular 277.8 and the Angle A $33^{\circ} 45'$ to find the Base and Hypothenufe.



LOGARITHMICALLY.

Here you have given the Perpendicular and Angle to find the Base and Hypothenufe. First look into Plate 2, and you will find as before, that the Perpendicular is Tangent of the Angle A, therefore,

As Tang. $\angle A$ $33^{\circ} 45'$	9.824892
Is to the Perpend. 277.8	2.443709

The other Part of the Proportion will be as before,

So

So is Radius 90 10.000000

To the Base 415.8 2.618817

Now by looking into Plate 2, you will find that the Hypothenufe is Secant of the Angle A; therefore you may either say,

As Radius, 90 10.000000

Is to the Base 415.8 2.618817

So is Secant of the $\angle A$ $33^{\circ} 45'$ 10.080153

To the Hypoth. 500 2.698970

or

As Tang. of the $\angle A$ $33^{\circ} 45'$ 9.824892

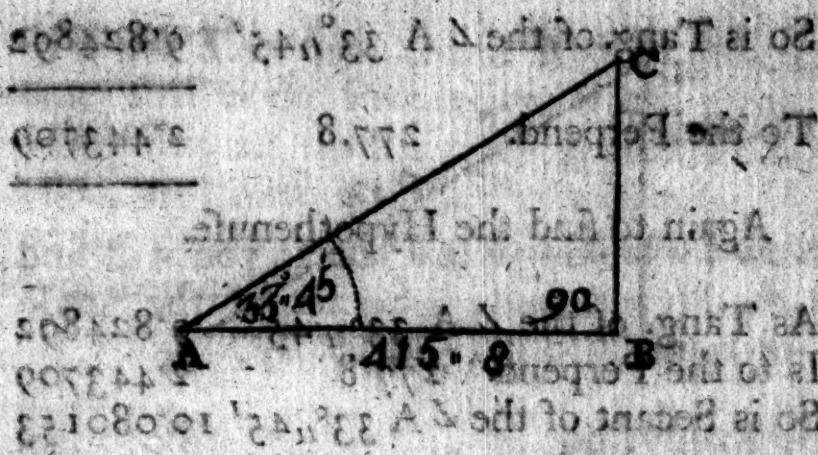
Is to the Perpend. 277.8 2.443709

So is Secant of the $\angle A$ $33^{\circ} 45'$ 10.080153

To the Hypoth. 500 2.698970

Question 3. In the right angled Triangle ABC are given the Base AB 415.8 and Angle A $33^{\circ} 45'$ to find the Perpendicular and Hypothenufe.

Logarithmically



LOGARITHMETICALLY.

Here you have the Base and Angles given to find the Perpendicular and Hypothenuſe. In the firſt Place, becauſe the Baſe is made Radius and you have it given, you muſt bring it in firſt, in the Proportion, agreeable to what is obſerved by way of Note, *Queſtion 1.* But a Side is required, therefore you muſt begin with an Angle, which Angle will be the Radius 90 by the Plate,

As Radius 90		10.000000
Is to the Baſe	415.8	2.618817

Now to find the Perpendicular, look into the Plate and you will find it Tangent of the Angle, A as before, therefore

So

So is Tang. of the $\angle A$ $33^{\circ}45'$ 9.824892

To the Perpend. 277.8 2.443709

Again to find the Hypothenufe.

As Tang. of the $\angle A$ $33^{\circ}45'$ 9.824892

Is to the Perpend. 277.8 2.443709

So is Secant of the $\angle A$ $33^{\circ}45'$ 10.080153

To the Hypoth. 500 2.698970

or

As Radius 90 10.000000

Is to the Base 415.8 2.618817

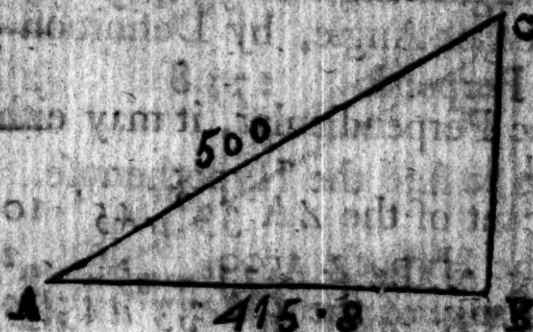
So is Secant of the $\angle A$ $33^{\circ}45'$ 10.080153

To the Hypoth. 500 2.698970

Question 4. In the right angled Triangle ABC , are given, the Base AB 415.8 and Hypothenufe AC 500 , to find the Angles and Perpendicular.

Logarithmetically.

TRIGONOMETRY.



LOGARITHMETICALLY.

Both the Base and Hypothenuſe are here given, but neither the Angles nor Perpendicular. Now as an Angle is to be found, a Side muſt come firſt in Proportion, which muſt be the Baſe, becauſe it is made Radius.

As the Baſe	415.8	2.618817
Is to Radius	90	10.000000

The Hypothenuſe will be the Secant of the $\angle A$, as before, therefore

So is the Hypoth.	500	2.698970
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To Secant of the $\angle A$	$33^{\circ}11'45''$	10.080153
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N. B.

TRIGONOMETRY.

N. B. This Angle subtracted from 90° will give the other Angle, by Definition 5.

For the Perpendicular, it may either be,

As Secant of the $\angle A$ $33^\circ 45'$	10.080153
Is to the Hypoth. 500	2.698970
So is Tang. of the $\angle A$ $33^\circ 45'$	9.824892

To the Perpend. 277.8	<u>2.443709</u>
-----------------------	-----------------

or

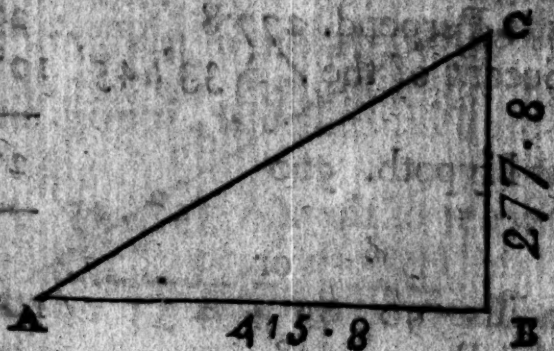
As Radius 90	10.000000
Is to the Base 415.8	2.618817
So is Tang. of the $\angle A$ $33^\circ 45'$	9.824892

To the Perpend. 277.8	<u>2.443709</u>
-----------------------	-----------------

Question 5. In the right angled Triangle A B C, there are given the Base A B 415.8, and Perpendicular B C 277.8 to find the Angles and Hypothense.

Logarithmically,

TRIGONOMETRY.



LOGARITHMETICALLY.

The Base and Perpendicular are here given but neither the Angles nor Hypothenuſe. As an Angle is to be found you muſt begin with a Side, and that Side muſt be the Baſe, therefore,

As the Baſe 415.8	2.618817
Is to Radius 90	10.000000
So is the Perpend. 277.8	2.443709

To the Tang. of the $\angle A$ 33 u 45'	9.824892
---	----------

N. B. This Angle ſubtracted from 90 Degrees will give the other, Definition 6.

For the Hypothenuſe you muſt begin with an Angle, therefore it may be,

H

As

As Tang. of the $\angle A$ $33^{\circ}11'45''$ 9.824892
 Is to the Perpend. 277.8 2.443709
 So is Secant of the $\angle A$ $33^{\circ}11'45''$ 10.080153

To the Hypoth. 500 2.698970

or

As Radius 90 10.000000
 Is to the Base 415.8 2.618817
 So is Secant of the $\angle A$ $33^{\circ}11'45''$ 10.080153

To the Hypoth. 500 2.698970

— o — o —

Questions to exercise the Second Variety.

1. In the right angled Triangle ABC are given the Hypothenuſe AC 817, and the Angle A $48^{\circ}11'45''$ to find the Base and Perpendicular.

Ans. The Base 538.7 and Perpend. 614.3

2. Given in the Triangle BGD , the Perpendicular GD 76 and the Angle B opposite thereto, $56^{\circ}11'17''$, quere the Base and Hypoth.

Ans. the Base 50.72 and Hypoth. 91.37

3. The

3. The Base XY 215 and Angle X $41^{\circ} 50'$, are given in the right angled Triangle XYZ , to find the Perpendicular and Hypothenuse.

Ans. The Perpend. 192.5 and Hypoth. 288.6

4. There is a Triangle whose Base is 318 and Perpendicular 200, required the Angles and Hypothenuse?

Ans. The Z opposite the Perpend. $32^{\circ} 10'$ and Hypoth. 375.7

5. Required the Angles and Perpendicular of the right angled Triangle EFG , when the Hypothenuse is 70 and Base 40.

Ans. The $\angle E$ $55^{\circ} 09'$ and Per. 57.45

6. In the Right angled Triangle BOI are given the Base 98 and Perpendicular 82, to find the Angles and Hypothenuse?

Ans. The $\angle B$ $39^{\circ} 38'$ and Hypoth. 128.5

VARIETY 3.

V A R I E T Y 3.

Perpendicular Radius.

IF the Perpendicular be made Radius, the Base becomes Tangent and Hypothenuſe Secant, ſee Plate 3. Fig. 3. where it is clear (from Def. 12 and 14) that as the Perpendicular is made Radius, the Base will be Tangent and Hypothenuſe Secant.

R U L E *

As Radius 90
Is to the Perpend.

So is Tangent of the $\angle$
opposite to the Baſe
To the Baſe.

And

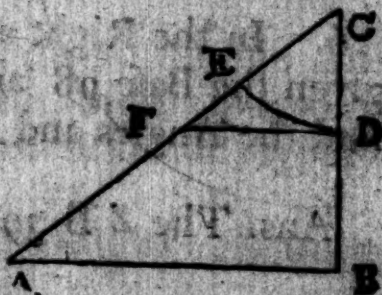
As Radius 90
Is to the Perpend.

So is Secant of the ſame $\angle$
To the Hypothenuſe.

And the contrary.

* D E M O N S T R A T I O N,

With the Centre C and any Radius CD, deſcribe an Arch DE, and erect the Perpendicular DF which, it is evident, will be the Tangent, and CF the Secant of the Arch DE, or Angle A to the Radius AD. And in ſimilar Triangles CDF, CBA it will be $CD : CB :: DF : BA :: CF : CA$. Q E D.



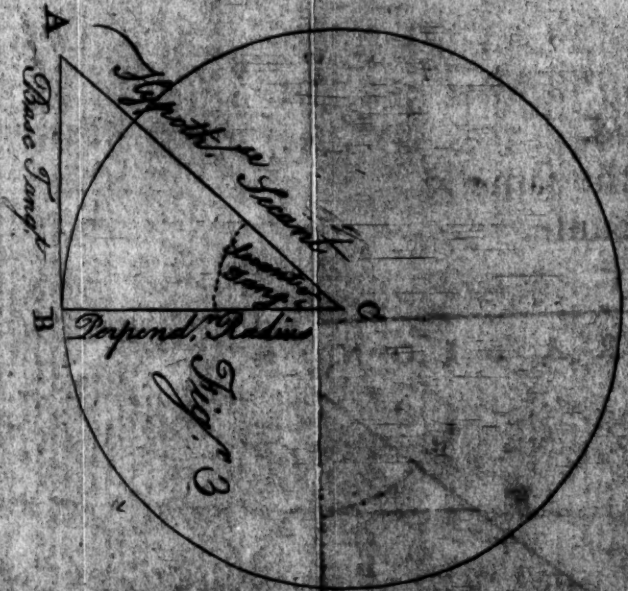
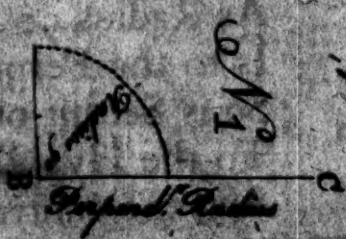
Question 1.

The third Variety of Perpend. Radius
 Plate 3
 of the 3^d Page 6



A
 Perpend. Radius
 B

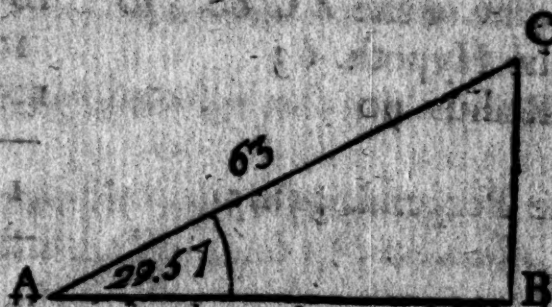
Perpend. Radius



A
 Perpend. Radius
 B

TRIGONOMETRY

Question 1. In the right angled Triangle ABC, there are given the Hypotenuse AC 63 and the Angle A $29^{\circ} 57'$ to find the Base and Perpendicular.



LOGARITHMETICALLY.

The Hypotenuse and Angles are here given to find the Base and Perpendicular. Turn to Plate 3, and you will soon perceive (No. 2 and 3) that your Proportion will run thus.

As Secant of the $\angle C$ $60^{\circ} 3'$	10.301687
Is to the Hypoth. 63	1.799341
So is Tang. of the $\angle C$ $60^{\circ} 3'$	10.239436
To the Base 54.59	1.737290

And

TRIGONOMETRY.

And for the Perpendicular, it is made Radi-
 us, therefore it must be, as shewn No. 1,
 Plate 9. But first take either Part of the last
 Proportion.

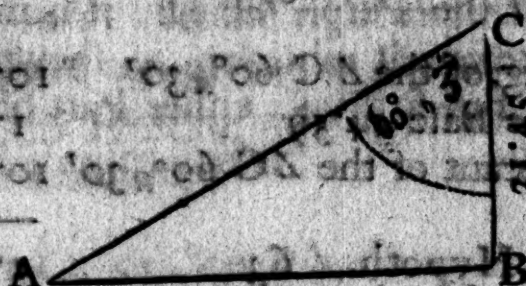
As Secant of the $\angle C$ $60^{\circ} 43'$	10.301687
Is to the Hypoth. 63	1.799341
So is Radius 90	10.000000
To the Perpend. 31.45	<u>1.497654</u>

Or, which will answer the same Purpose.

As Tang. of the $\angle C$ $60^{\circ} 43'$	10.239436
Is to the Base 54.59	1.737090
So is Radius 90	10.000000
To the Perpend. 31.45	<u>1.497654</u>

Question 2. In the right angled Triangle
 A B C, are given the Perpendicular B C
 31.45 and Angle A $29^{\circ} 57'$, required the Base
 and Hypothenuse.

Logarithmically



LOGARITHMETICALLY.

The Angles and Perpendicular are here given and the Base and Hypothenufe required.

To find the Base, it will be,

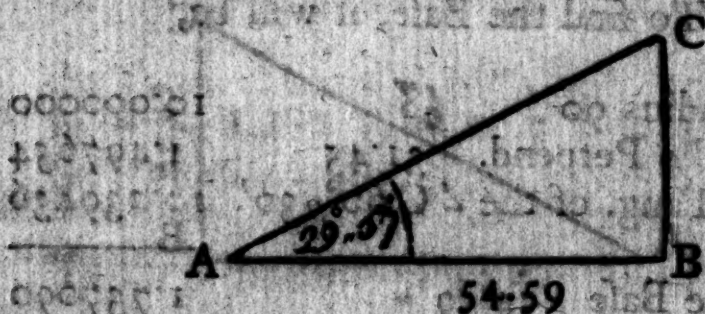
As Radius 90		10.000000
Is to the Perpend.	31.45	1.497654
So is Tang. of the $\angle C$	60.30	10.239436
To the Base	54.59	1.737090

And for the Hypoth.

As Radius 90		10.000000
Is to the Perpend.	31.45	1.497654
So is Secant of the $\angle C$	60.30	10.30887
To the Hypoth.	63.80	1.799541

As Tang. of the $\angle C$ $60^{\circ} 11' 30''$ 10.239436
 Is to the Base 54.59 1.737090
 So is Secant of the $\angle C$ $60^{\circ} 11' 30''$ 10.301687
 To the Hypoth. 63 1.799341

Question 3. In the right angled Triangle ABC there are given the Base 54.59 and the Angle A $29^{\circ} 11' 57''$, required the Perpendicular and Hypothenufe?



LOGARITHMICALLY.

As Tang. of the $\angle C$ $60^{\circ} 11' 30''$ 10.239436
 Is to the Base AB 54.59 1.737090
 So is Secant of the $\angle C$ $60^{\circ} 11' 30''$ 10.301687
 To the Hypoth. 63 1.799341

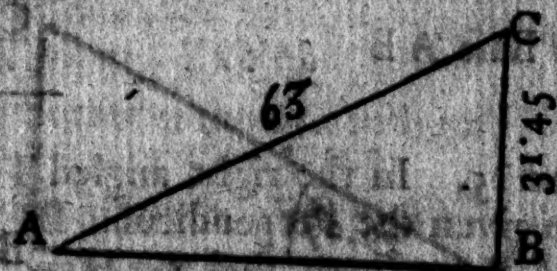
To

TRIGONOMETRY.

To find the Perpendicular.

As Secant of the $\angle C$ $60^{\circ}11'03''$	10.301687
Is to the Hypoth. 63	1.799341
So is Radius 90	10.000000
To the Perpend. 31.45	1.497654

Question 4. In the right angled Triangle ABC, there are given the Perpendicular BC 31.45 and the Hypothenufe 63 to find the Base and Angles.



LOGARITHMetically.

For the Angle C.

As the Perpend. BC 31.45	1.497654
Is to Radius 90	10.000000
So is the Hypoth. AC 63	1.799341
To Secant of the $\angle C$ $60^{\circ}11'30''$	10.301687

I

To

TRIGONOMETRY.

To find the Base.

As Radius 90 10.000000
 Is to the Perpend. BC $31^{\circ}45'$ 1.497654
 So is Tang. of the $\angle C$ $60^{\circ}30'$ 10.239436

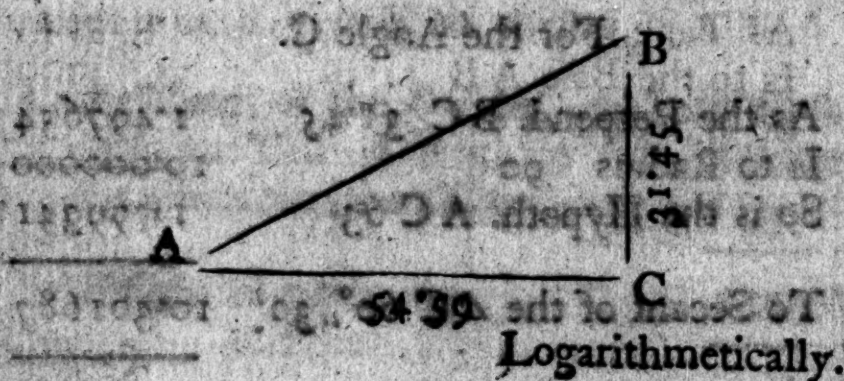
To the Base AB $54^{\circ}59'$ 1.737090

or

As Secant of the $\angle C$ $60^{\circ}30'$ 10.301687
 Is to the Hypoth. AC 63 1.799341
 So is Tang. of $\angle C$ $60^{\circ}30'$ 10.239436

To the Base AB $54^{\circ}59'$ 1.737090

Question 5. In the right angled Triangle ABC are given the Perpendicular BC $31^{\circ}45'$ and Base AB $54^{\circ}59'$ to find the Hypothenufe and Angles.



TRIGONOMETRY.

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LOGARITHMICALLY.

To find the Angle C.

As the Perpend. BC	31'45"	1.497654
Is to Radius 90		10.000000
So is the Base AB	54'59"	1.737090
To Tang. of the $\angle C$	$60^{\circ} 11' 30''$	10.239436

To find the Hypothenufe.

As Radius 90		10.000000
Is to the Perpend. 31'45"		1.497654
So is Secant of the $\angle C$	$60^{\circ} 11' 30''$	10.301687
To the Hypoth. AC	63	1.799341

or

As Tang. of the $\angle C$	$60^{\circ} 11' 30''$	10.239436
Is to the Base AB	54'59"	1.737090
So is Secant of the $\angle C$	$60^{\circ} 11' 30''$	10.301687
To the Hypoth. AC	63	1.799341

Questions

Questions to exercise the Third Variety,

1. In the right angled Triangle ABC there are given the Hypothenufe AC 180 and Angle A $45^{\circ}10'$ to find the Perpendicular and Base.

Ans. The Perpend. 127.3 and Base 127.3

2. There is a Triangle the Perpendicular of which is 97 and the Angle opposite thereto $29^{\circ}55'$, quere the Hypothenufe and Base.

Ans. The Hypoth. 194.5 and Base 168.6.

3. Given in the right angled Triangle XOZ , the Base XO 58 and the Angle Z 60° , to find the Perpendicular and Hypothenufe.

Ans. The Per. 33.48 and Hypoth. 66.97.

4. Required the Angles and Base of the right angled Triangle POQ , the Hypothenufe of which is 12 and Perpendicular 8.

Ans. The $\angle P$ $41^{\circ}49'$; $\angle Q$ $48^{\circ}11'$
and Base 8.944.

5. The

TRIGONOMETRY.

5. The Perpendicular of the right angled Triangle AOY is given equal to 75 and Base 60, required the Hypothenufe AY , and Angles.

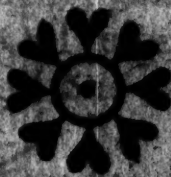
Ans. Hypoth. AY 96.06; $\angle A$ $51^{\circ}14'$ and $\angle Y$ $38^{\circ}46'$.

6. In the right angled Triangle ABC are given the Angle A $34^{\circ}36'$ and Hypothenufe AC 120, to find the Base and Perpendicular.

Ans. Base 98.78 and Perpend. 68.14.

7. Given in the right angled Triangle STU , the Perpendicular TU 120 and Base ST 130, to find the Angles and Hypothenufe.

Ans. Hypoth. 176.9; $\angle S$ $42^{\circ}43'$ and $\angle U$ $47^{\circ}17'$.





TRIGONOMETRY.

Oblique Plain Trigonometry.

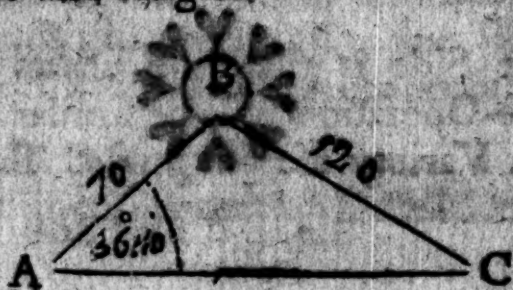
SECTION 4.

IN Definition 5th. it is said, that an oblique angled plain Triangle is such as has no right Angle. As in right angled Triangles there are 6 Cases, so in oblique there are 4, which I shall now proceed to explain.

N. B. Oblique angled plain Triangles, as well as right angled ones, have their Sides proportional to each other, as the Sines of their opposite Angles, but when there is not a proper Data given for such a Proportion, other Rules are observed for bringing out the Solutions, which I shall point out in their proper Places.

C A S E I.

In the oblique angled plain Triangle ABC , there are given the Side BC 120, the Side AB 70 and the Angle A $36^{\circ}40'$; required the other Side and Angles.



Geometrically.

GEOMETRICALLY

—Draw B A and assume any Point thereon, as A, with the Chord of 60° in your Compasses, and one Foot in A, draw the Arch b a, and set off thereon $36^\circ, 40'$ through a, draw A B and lay off A B equal 70, then with B C equal 120 in your Compasses and one foot in B, cross with the other, the Line A C, as in C; draw C B, and you will have the Triangle required.

The Angles may be found by taking the Chord of 60° in your Compasses, and with one Foot in C and B describe the Arches a b and c d, which take in your Compasses and apply to the Line of Chords, give their respective measure.

—The Side B C being taken in your Compasses and applied to a Line of equal Parts gives its Length.

LOGARITHMETICALLY

As each Side is the Sine of its opposite Angle, the Operation will be similar to those in the 1st. Variety of right angled Triangles.

As

As the Side BC 120 2.079181
 Is to the Sine of the $\angle A$ $36^{\circ}40'$ 9.776090
 So is the Side AB 70 1.845098

To the Sine of the $\angle C$ $20^{\circ}24'$ 9.542007

The Angle C added to the Angle A, and
 subtracted from 180 Degrees, gives the re-
 maining Angle B.

Again; for the Side AC,

As the Sine of the $\angle C$ $20^{\circ}24'$ 9.542007
 Is to the Side AB 70 1.845098
 So is the Sine of the $\angle B$ $122^{\circ}56'$ 9.923919

To the Side AC 168.7 2.227010

As the Sine of the $\angle A$ $36^{\circ}40'$ 9.776090
 Is to the Side BC 120 2.079181
 So is the Sine of the $\angle B$ $122^{\circ}56'$ 9.923919

To the Side AC 168.7 2.227010

N. B. In taking the Sine of the obtuse
 Angle B, remember that by Definition 7th. the
 Sine

Sine of $122^{\circ} 56'$ is also the Sine of $57^{\circ} 4'$, the one being the Supplement of the other, therefore from the Table of Sines you are to take the Sine of $57^{\circ} 4'$, because every right Sine is the Sine of two Arches, the one being the Supplement of the other.

C A S E 2.

In the oblique angled plain Triangle ABC are given, the Angle A $36^{\circ} 40'$, B $122^{\circ} 56'$, C $20^{\circ} 24'$, and the Side AC 168.7, required the Sides AB and BC.



Draw AC, and lay off 168.7 thereon from a Scale of equal Parts.

With the Chord of 60 in your Compasses draw the Angle A, and lay off thereon $36^{\circ} 40'$, do the same with the Angle C, and lay off $20^{\circ} 24'$, draw AB and BC and the Triangle is completed.

Logarithmically.

ARITHMETICALLY.

To find the Side A B.

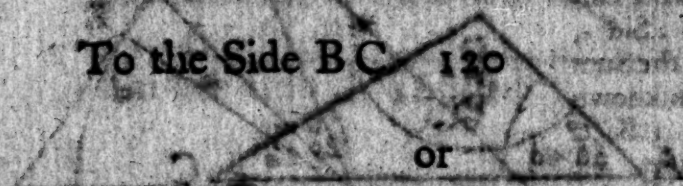
As the Sine of the $\angle B$ $122^{\circ} 56'$ 9.923919
 Is to the Side A C 168.7 2.227115
 So is the Sine of the $\angle C$ $20^{\circ} 24'$ 9.542007

To the Side A B 70 1.845203

Again; to find B C.

As the Sine of the $\angle B$ $122^{\circ} 56'$ 9.923919
 Is to the Side A C 168.7 2.227115
 So is the Sine of the $\angle A$ $36^{\circ} 40'$ 9.776090

To the Side B C 120 2.079286



As the Sine of the $\angle C$ $20^{\circ} 24'$ 9.542293
 Is to the Side A B 70 1.845203
 So is the Sine of the $\angle A$ $36^{\circ} 40'$ 9.776090

To the Side B C 120 2.079000

CASE 3.

Two Sides and the contained Angle of an oblique angled plain Triangle, being given to find the other Angles and Side.

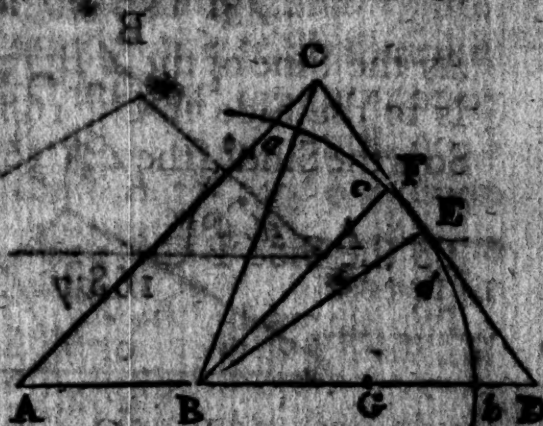
RULE

TRIGONOMETRY

As the Sum of the two Sides
Is to their Difference
So is the Tang. of half the Sum of the
unknown Angles
To Tang. of half their Difference

* DEMONSTRATION.

Suppose ABC the Triangle, make $BD = BC$, and $BG = AB$, then is $GD =$ the Difference of the 2 Sides, and $CD =$ the Sum of the two unknown Angles (Prop. 5 Sect. 2.) Therefore, let fall the Perpendicular BE , and with that



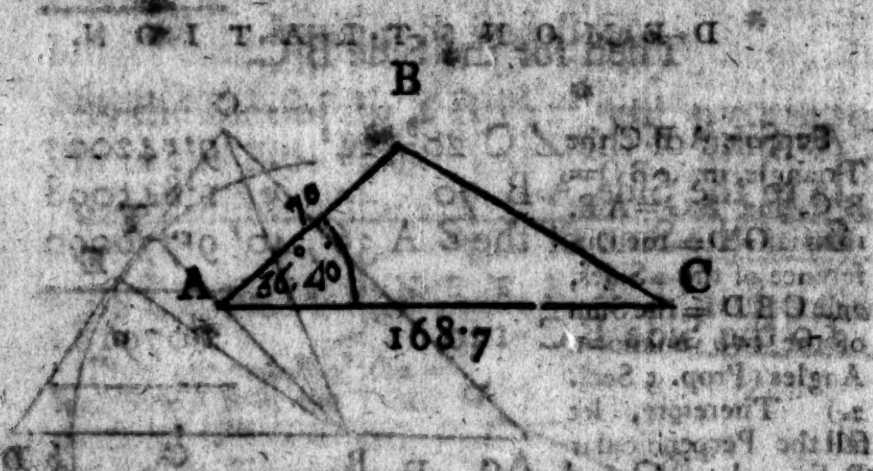
as a Radius, describe the Arch, ED and draw CD , then will ED be $=$ the Tangent of half the Sum of the two unknown Angles. Draw BF parallel to AC . Then, per similar Triangles ABC and EBF (Prop. 1 Sect. 5) we have $AB : BC :: BE : EF$.

As $AG : GD :: DE : EF$. Q. E. D. But half the Difference subtracted from half the Sum gives the lesser Angle, and added to half the Sum gives the greater.

Suppose $a =$ the Sum of the two Angles, and $c =$ the greater. Now the Line BE cuts a into two equal Parts, $a/2$ and $a/2$, make $Ed = c/2$, then will FD be the whole Difference, and the Line ED will be the Tangent of half the Sum. If ED be subtracted from the half Sum $a/2$, it will give the lesser $a/2$, and if added $c/2$ to $a/2$, gives the greater $c/2$. Q. E. D.

To half the Sum of the two Angles, add half their Difference, the Sum will be the greater Angle and subtracting their half Difference from the half Sum, gives the lesser Angle.

In the oblique angled plain Triangle ABC, the Side AB 70, AC 168.7 and the Angle between them $36^{\circ} 40'$ are given, to find the other two Angles and Side.



GEOMETRICALLY.

Draw AB and set off 70 from a Scale of equal Parts.

With the Chord of 60 in your Compasses describe the Angle A and set off thereon $36^{\circ} 40'$. Draw AC equal 168.7 and join CB; which completes the Triangle.

Logarithmically.

TRIGONOMETRY

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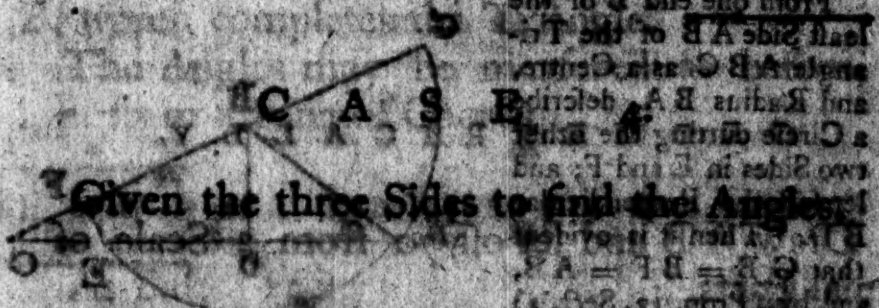
LOGARITHMICALLY.

As the Sum of two Sides 238.7 2.377852
Is to their Difference 108.9 1.0394317
So is the Tangent of half the Sum 10.479695
To the Tangent of half the Difference 10.000160

Then for the Side B C longer Side
As Sine of the $\angle C$ 20.24' 9.542007
Is to the Side A B 70 1.844818
So is the Sine of the $\angle A$ 36.40' 9.776090

To the Side B C 120 1.816702

Given the three Sides to find the Angles



RULE

TRIGONOMETRY

RULE

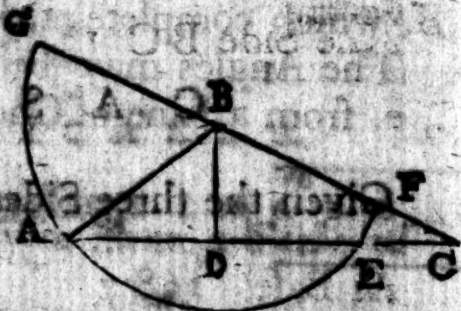
As the Base or largest Side
Is to the Sum of the other Two Sides
So is the Difference of those Sides

To the Difference of the Segments of
the Base, made by a Perpendicular let fall
from the vertical Angle upon the said Base.
Then half the Difference added to half the
longer Side, gives the greater Segment, and
subtracted from it, gives the lesser.

In $\triangle ABC$ the Side AB is to the Side AC as the Sum of the Sides AB and BC is to the Sum of the Segments AD and DC of the Base AC .

DEMONSTRATION

From one end B of the
least Side AB of the Tri-
angle ABC as a Centre,
and Radius BA , describe
a Circle cutting the other
two Sides in E and F ; and
let fall the Perpendicular
 BD . Then it is evident
that $GB = BF = AB$,
and by (Prop. 13. Sect. 2)

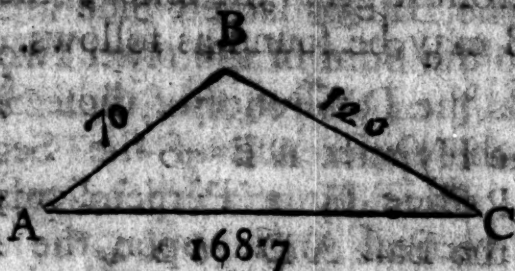


$AD = DE$, and consequently $EC = CD - DA$, $FC = CB -$
 BA , and $GC = CB + BA$. But Prop. 9 Sect. 2. $AC:$
 $CG :: FC: CE$ that is, $AC: CB + BA :: CB - BA:$
 $CD - DA$ QED.

And that the half Sum of two Quantities increased and de-
minished by their half Difference, gives the greater and less
Quantities respectively, was proved in the last Case.

TRIGONOMETRY.

In the oblique angled plain Triangle ABC are given AB 120, AC 168.7 and BC 70, to find the Angles A, B and C.



GEOMETRICALLY.

Draw the Line AC and lay off thereon 168.7 from a Scale of equal Parts. From the same Scale take 70 in your Compasses and one Foot in A describe an Arch, then take 120 and with one Leg in C describe with the other, as second Arch cutting the Former in B, which completes the Triangle.

The Angles may be measured as before, i, e, from a Line of Chords.

LOGARITHMATICALLY.

As Base 168.7 2.227115

Is to the Sum of the other two } 2.278754
Sides 190

So is the Diff. of those Sides 50 1.698970

To the Diff. of the Seg. of the } 1.758609
Base 56.31

Now

Now half the Difference of the Segments is 28.15 which added to $84.35 =$ half the longer Side, gives the greater Segment, and subtracted from it gives the lesser; therefore the Angle B may be found as follows.

As the Hypoth. A B	70	1.845098
Is to Radius	90	10.000000
So is the Base A D	56.2	1.749736

To the Sine of the $\angle B$	$53^{\circ} 24'$	9.904638
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The Angle B subtracted from 90 Degrees leaves the $\angle A$, or it may be found by making the Base Radius, thus:

As the Base A D	56.2	1.749736
Is to Radius	90	10.000000
So is the Hypoth. A B	70	1.845098

To the Secant of the $\angle A$		10.095362
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Questions

TRIGONOMETRY.

81

Questions to exercise the Four Cases of oblique angled plain Trigonometry.

1. In the oblique angled plain Triangle ABC are given the Side AB 80, the Side BC 137 and the Angle C $42^{\circ} 10'$, required the Side AC and the other Angles.

Ans. The Side AC 187.2; the $\angle A$ $40^{\circ} 15'$ and $\angle B$ $117^{\circ} 35'$.

2. Given in the oblique angled plain Triangle DGH, the $\angle D$ $27^{\circ} 40'$, the Angle G $107^{\circ} 26'$, Angle H $44^{\circ} 54'$, and the Side DH 120, required the other two Sides,

Ans. GH 58.4 and the Side DG 88.78.

3. There is an oblique angled plain Triangle XOY, the Side XO 410, XY 600 and the Angle between them $38^{\circ} 50'$, required the other Angles and Sides.

Ans. $\angle Y$ $42^{\circ} 30'$; $\angle O$ $98^{\circ} 40'$ and the Side YO 381.

4. In

4. In the oblique angled plain Triangle ABC are given the Side AB 110, BC 80 and AC 150, required the Angles.

Ans. The $\angle A$ $31^{\circ}17'$; $\angle C$ $45^{\circ}35'$; }
 $\angle B$ $103^{\circ}08'$.

5. Given in the oblique angled plain Triangle CDE the Side CD 127, CE 200 and the $\angle D$ $109^{\circ}0'$, required the other Side and Angles.

Ans. The Side DE 118.5; $\angle C = 34^{\circ}6'$ }
 and $\angle E$ $36^{\circ}54'$.

6. The Side AB 69, and AC 90, and the Angle between them $50^{\circ}45'$ are given in the oblique angled Triangle ABC , to find the other Side and Angles.

Ans. The Side BC 70.73; $\angle B$ $80^{\circ}10'$ }
 and $\angle C$ $49^{\circ}44'$.

Heights and Distances.

C H A P. 2.

S E C T I O N 5.

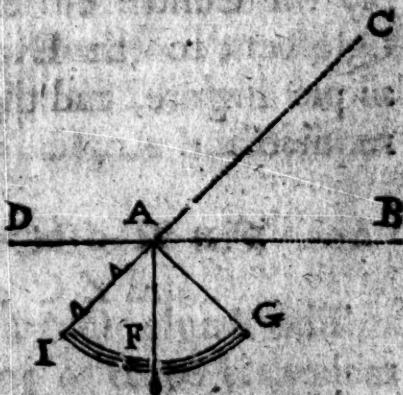
THE Object to be measured may be either accessible or inaccessible.

Accessible Lines are measured by the Application of some certain Measure to them, as a Foot, a Yard, a Chain &c. a certain Number of Times.

Inaccessible Lines are measured by Means of Angles: Angles are taken and then the Inaccessible Lines are found by Trigonometrical Calculations.

To take an Angle of Altitude with the Quadrant.

Suppose C to be any Object as the Top of a Mountain, Tree or House, and you want to know what Angle it makes with the Horizon from the Point A. First fix the Centre of your Quadrant at A,



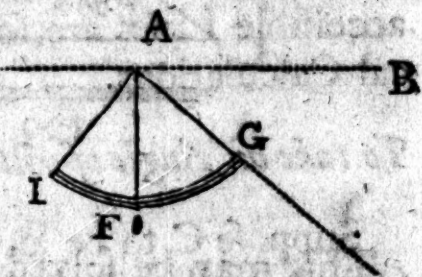
then move it up and down till such Times as you observe through the Sights the Object C, then

then observe what Number of Degrees FG is cut by the Plum AF , and that will be the Measure of the Angle CAB as required. The truth of this may easily be demonstrated, thus:

First produce BA to D , then is DAF equal to a right Angle (by Prop. 1. Sect. 2.) and IFG is known to be equal to a right Angle. But $DAI + IAF = FAG + IAF$. Therefore $FAG = DAI$ (because IAF is common to both) which also is $= CAB$ (Prop. 2. Sect. 2.) Q E D.

To take an Angle of Depression.

This is done in the manner as the last, only here you must apply the Centre of the Quadrant to the Eye,



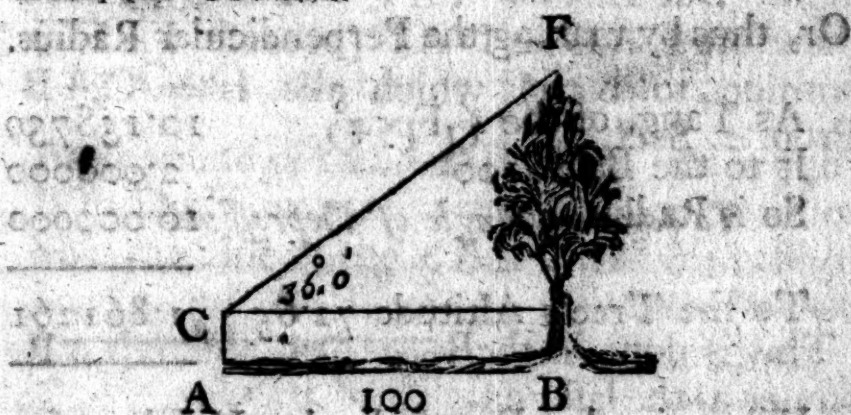
as per Figure, and the Arch FG is the Angle required.

DEMONSTRATION.

The Angle GAI which the Instrument makes, is equal to 90° and BAF is equal to 90° . (Prop. 1. Sect. 2.) But $GAF + FAI = BAI + FAI$, therefore $GAF = BAI$. Q E D. Wanting

HEIGHTS and DISTANCES. 85

Wanting to know the Height of a large Elm Tree, which stood upon a horizontal Plain, I measured from its bottom B in a direct Line just 100 Yards to A, there I took the Angle of Elevation of the Top, which I found just $36^{\circ} 10'$, I demand the Height of the said Tree, the Quadrant being four Feet from the Ground.



This may be solved by the 1st. Variety.

As the Sine of the $\angle F$ 54	9.907958
Is to the Base A B 100	2.000000
So is the Sine of the $\angle C$ 36°	9.769219

To the Tree's Altitude 72.65	1.861261
------------------------------	----------

To which add four Feet the Height of the Instrument, and it gives the true Altitude 76.65 Yards. Or,

66 HEIGHTS and DISTANCES

Or, thus by making the Base Radius.

As Radius 90	10.000000
Is to the Base A B 100	2.000000
So is the Tang. of the $\angle C$ 36°	9.861261
To the Tree's Altitude 72.65	1.861261

Or, thus by making the Perpendicular Radius.

As Tang. of the $\angle F$ 54°	10.138739
Is to the Base 100	2.000000
So is Radius 90	10.000000
To the Tree's Altitude 72.65	1.861261

What is the Perpendicular Height of a Hill, whose Angle of Elevation taken at the Bottom of it was 46° , and 100 Yards farther off on a Level with the Bottom of it, was 31° .



The

The external Angle BCA made less by the Angle D is equal to the Angle DBC (per Prop. 5. Sect. 2.)

$$\begin{array}{r} 46^\circ \\ 31^\circ \\ \hline 15^\circ \end{array} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Subtract.}$$

By Case 1. of oblique Trigonometry.

As the Sine of the $\angle DBC$ 15° 9'412996

Is to DC 100 2'000000

So is the Sine of the $\angle BDC$ 31° 9'711839

To the Side BC 199 2'298843

Then to find BA the Height of the Hill, proceed by the 1st. Variety of right angled Triangles.

As Radius 90 10'000000

Is to the Hypoth. BC 199 2'298843

So is the Sine of the $\angle C$ 46° 9'856934

To the Perpend. BA 143'14 2'155777

It

88 HEIGHTS and DISTANCES.

It may also be found by the 2d. and 3d. Varieties, thus:

As the Secant of the $\angle C$ 46°	10.158229
Is to the Hypoth. BC 199	2.298843
So is the Tang. of the $\angle C$ 46°	10.015163
To the Perpend. BA 143.14	2.155777

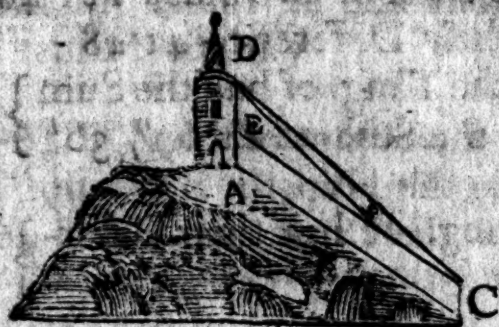
and

As the Secant of the $\angle B$ 44°	10.143066
Is to the Hypoth. 199	2.298843
So is Radius 90	10.000000
To the Perpend. 143.14	2.155777

An Obelisk standing on the Top of a Delivity, I measured from its Bottom a Distance of 40 Feet, and then took the Angle formed by the Plain and a Line drawn to the Top 41° ; and going on in the same Direction 60 Feet farther, the Angle there was $23^\circ 45'$ the Height of the Instrument being 5 Feet. What was the Height of the Obelisk.

You

HEIGHTS and DISTANCES. 89



You must first find the Side BD which you may do by Case 1. of oblique angled Trigonometry.

As Sine of the $\angle D 17^{\circ} 15'$ 9.472086

Is to BC 60 1.778151

So is the Sine of the $\angle C 23^{\circ} 45'$ 9.605032

To the Side BD 81.48 1.911097

Now in the Triangle EBD you have two Sides $EB = 40$, $BD = 81.48$ and the Angle contained between them 41° to find the Side DE .

In the first place work for the other two Angles by Case 3d. of oblique Trigonometry, thus:

M

As

90 HEIGHTS and DISTANCES

As the Sum of two Sides $121^{\circ}48' 2^{\circ}08'42''19$
 Is to their Difference $41^{\circ}48' 1^{\circ}61'78''39$
 So is the Tang. of half the Sum } $10^{\circ}42'72''62$
 of the 2 unknown $\angle$ s $69^{\circ}11'30''$ }

To Tang. of half their Differ- } $9^{\circ}96'08''82$
 ence $42^{\circ}11'25''$ }

111155 Greater $\angle$ E

27115 Lesser $\angle$ D

Then the Side DE may be found by Case
 1st. of oblique Trigonometry.

As the Sine of the $\angle$ E 111155 $9^{\circ}96'74''20$
 Is to the Side BD $81^{\circ}48'$ $1^{\circ}911'09''7$
 So is the Sine of the $\angle$ B 41° $9^{\circ}81'69''43$

To the Side DE $57^{\circ}63'$ $1^{\circ}760'62''0$

AE 5

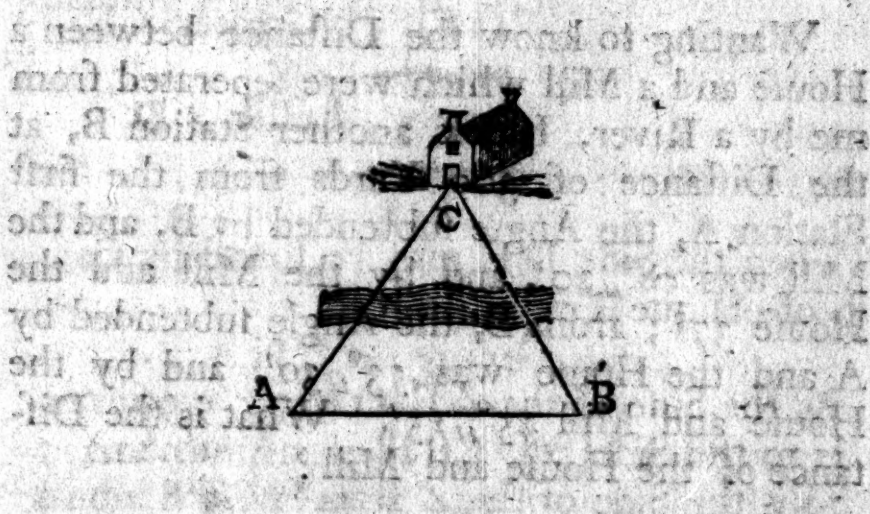
AD = $62^{\circ}63'$ the Height required.

Being on the Side of a River and wanting
 to know the Distance of a House which stood
 on

HEIGHTS and DISTANCES.

91

on the other Side, I measured 200 Yards in a right Line by the Side of the River and found the two Angles at each End of this Line formed by the other End and the House were $A = 73^{\circ} 15'$ and $B = 68^{\circ} 2'$. What was the Distance between each Station and the House?



For the Side BC, by Case 1st. oblique Trigonometry.

As the Sine of the $\angle C$ $38^{\circ} 43'$	9.796206
Is to the Side AB 200	2.301030
So is the Sine of the $\angle A$ $73^{\circ} 15'$	9.981171

To the Side BC 306.19	2.485995
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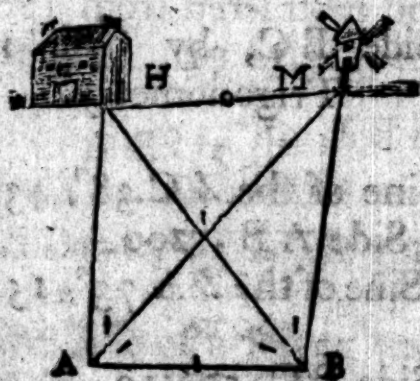
For

92 HEIGHTS and DISTANCES.

For the Side A C, by the same.

As Sine of the $\angle C$ $38^{\circ} 43'$	9.796206
Is to the Side A B 200	2.301030
So is the Sine of the $\angle B$ $68^{\circ} 2'$	9.967268
To the Side A C 296.54	2.472092

Wanting to know the Distance between a House and a Mill which were seperated from me by a River, I took another Station B, at the Distance of 300 Yards from the first Station A, the Angle subtended by B, and the Mill was $58^{\circ} 20'$, and by the Mill and the House 37° ; from B, the Angle subtended by A and the House was $53^{\circ} 30'$, and by the House and Mill $45^{\circ} 15'$. What is the Distance of the House and Mill?



In

HEIGHTS and DISTANCES. 93

In the Triangle ABH, you have all the Angles given, for the Angle HBA = $53^{\circ} 30'$ and the Angle MAB and HAM = $58^{\circ} 20'$ and $37^{\circ} 00'$, respectively, which are equal $95^{\circ} 20' =$ the $\angle$ HAB, therefore the Angle HAB added to the Angle HBA and subtracted from 180 Degrees will give the remaining Angle AHB, thus:

$$\begin{array}{r} 95^{\circ} 20' \\ 53^{\circ} 30' \\ \hline 148^{\circ} 50' \\ 180^{\circ} 00' \\ \hline AHB = 31^{\circ} 10' \end{array}$$

And the Angle HBA + HBA gives the Angle ABM = $98^{\circ} 45'$ which added to the Angle A and subtracted from 180° gives the remaining Angle, thus:

$$\begin{array}{r} ABM 98^{\circ} 45' \\ A 58^{\circ} 20' \\ \hline 157^{\circ} 5 \\ 180^{\circ} 00' \\ \hline \angle AMB 22^{\circ} 55' \end{array}$$

Then

94 HEIGHTS and DISTANCES

Then for the Line A H.

As the Sine of the $\angle AHD$ $31^{\circ} 10'$ 9.713935
 Is to the Side A B 300 2.477121
 So is the Sine of the $\angle ABH$ $53^{\circ} 08'$ 9.905179
 To the Side A H 465 2.668365

For the Side A M, by the same.

As the Sine of the $\angle AMB$ $22^{\circ} 25'$ 9.590386
 Is to A B 300 2.477121
 So is the Sine of the $\angle ABM$ $98^{\circ} 45'$ 9.994915
 To the Side A M 761.46 2.881650

Now in the Triangle A H M you have two Sides and the $\angle$ between them, to find the other Side and Angles, you must therefore proceed by Case 3 of oblique Trigonometry.

	761.46	180° 00'
	465.91	37° 00' $\angle MAH$
The Sum	1227.37 of 2 S.	143° 00' sum of 2 $\angle$ s
Their	295.55 Diff.	71° 30' Half Do.
		As

HEIGHTS and DISTANCES. 95

As the Sum of 2 Sides 1227.37 3.088890
 Is to the Difference 295.55 2.460631
 So is Tang. of half the Sum of }
 the 2 unknown $\angle$ s 71° 30' } 10.475480

To Tang. of half their Diff. }
 35.17 } 9.847221

106.37 Greater $\angle$ H

36.21 Lesser $\angle$ M

Next for the required Distance.

As the Sine of the $\angle$ AMH 36° 21' 9.772847
 Is to the Side A H 465.9 2.668365
 So is the Sine of the $\angle$ HAM 37.0' 9.779463

To the Side H M 474.2 2.675981

PROMISCUOUS

Promiscuous Questions.

1. **I**N the right angled Triangle ABC there are given the Hypothenufe AC 100, the Angle A $33^{\circ}45'$ to find the Perpendicular and Base.

Ans. Perpend. 55.56 and Base 83.15 .

2. In the oblique angled plain Triangle CDE the Side $CD = 50$, $CE = 80$ and the Angle DCE which is contained between them $= 38^{\circ}$, quere the other Side and Angles.

Ans. The $\angle E$ $37^{\circ}10'$, $\angle D$ $104^{\circ}50'$ }
and the Side 50.59 .

3. There is a Triangle ABC whose Base AB is 80 and Perpendicular BC 100, what are the Angles and Hypothenufe?

Ans. The $\angle A$ $51^{\circ}20'$, C $38^{\circ}40'$ and
Hypoth. 128.

4. Wantin

PROMISCUOUS QUESTIONS. 97

4. Wanting to know the Breadth of a River, I measured 150 Yards in a straight Line close by one Side of it, and at each End of this Line I found the Angles subtended by the other End, and a Tree close by the other Side of the River, to be 63° and $75^{\circ} 30'$. What is the Perpendicular Breadth.

Ans. 130.2 Yards.

5. There is an oblique angled plain Triangle whose three Sides AB, BC and CA are 60, 100 and 120 respectively, required the Angles.

Ans. $\angle C 29^{\circ} 56'$, $\angle B 93^{\circ} 49'$ and $\angle A 56^{\circ} 15'$

6. In the right angled plain Triangle DEF are given DE the Base 76, and Hypothenufe DF 90, quere the Angles and Perpendicular.

Ans. $\angle F 57^{\circ} 37'$, $\angle D 32^{\circ} 23'$ and Perpendicular 48.09

N

4. In

7. In the oblique angled plain Triangle A B C, are given the $\angle A 37^\circ$, $B 138^\circ$ and $C 25^\circ$ and the Side A B 109; required the other two Sides.

Ans. The Side B C 117.1 and A C 172.9.

8. From a Ship at Sea I observed a Point of Land to bear E by S, and after sailing N E 12 Miles I set out again and found its bearing to be S E by E. How far was the last Observation made from the Point of Land.

Ans. 26.07 Miles.

9. Given in the right angled Triangle, A B C the Hypothenuse A C 650 and Perpendicular B C 500, I demand the Angles and Base.

Ans. The $\angle A 50^\circ 17'$, $\angle C 39^\circ 43'$ and Base A B 415.4.

16. Wanting to know the Height of a Summer House, which stood upon a level Plat of Ground, I measured 60 Yards from its Bottom

in

PROMISCUOUS QUESTIONS. 99

In a direct Line, and there took the Angle of Elevation of the Top, which I found to be $2^{\circ} 0'$, and going 80 Yards farther, found the Angle there only 28° ; quere the Height of the said Summer House the Quadrant being placed Feet above the Ground?

Ans. 76.76 Yards.

11. There is a Triangle ABC, whose Hypothenufe AC is 240, and Base AB 220; what are the Angles.

Ans. The $\angle A$ $36^{\circ} 2'$, and $\angle C$ $53^{\circ} 58'$.

12. From the Top of a Ship's Mast, which was 80 Feet above the Water, the Angle of Depression of another Ship's Hull upon the Water at a Distance is 20° ; what is their Distance.

Ans. 219.79 Feet.

F I N I S.

to the line, and there took the Angle of
 of the Top, which I found to be
 the same as the Angle of the
 there only 28 degrees the height of the
 summer House the constant being placed
 above the Ground.

and the same

There is a Table of the whole H.
 and the A. B. C.

ERRATA.

- Page 14 Line 6 for unequal, read equal
- Do, Line 7 for unequal, read equal.
- Page 36 Line 3 for A B read C B.
- Page 38 Line 16 for No. 3 read No. 2.
- Page 45 Line 15 for No. 1 read No. 3.
- Page 88 *Dyle* Section 5.



and the same

There is a Table of the whole H.